ECS 315: In-Class Exercise # 10

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **09** / **10** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

$P(\{\omega\}) = \frac{1}{6}$ for $\omega = 1, 2, 3, \ldots, 6$

1. Consider a random experiment in which you roll a six-sided fair dice (whose faces are numbered 1-6). We define the following random variables from the outcomes of this experiment:

$$X(\omega) = \omega \quad \text{and} \quad Y(\omega) = 1 + \left((\omega-2)(\omega-3)(\omega-5)(\omega-8)\right).$$

a. Find $P[X=5]$.

$X(\omega) = 5$ when $\omega = 5$ $\Rightarrow P[X=5] = P(\{5\}) = \frac{1}{6}$

b. Find $P[Y=1]$.

$Y(\omega) = 1$ when $1 + \left((\omega-2)(\omega-3)(\omega-5)(\omega-8)\right) = 1$

not in Ω

$\omega = 2, 3, 5, \cancel{8}$

$\Rightarrow P[Y=1] = P(\{2\}) + P(\{3\}) + P(\{5\}) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$

$P(\{\omega\}) = \frac{1}{10}$ for $\omega = 1, 2, \ldots, 10$

2. Consider a random experiment in which you roll a 10-sided fair dice (whose faces are numbered 0–9). Define a random variable Z from the outcomes of this experiment by

$$Z(\omega) = (\omega-7)^2.$$

a. Find $P[Z=4]$.

$Z(\omega) = 4$ when $(\omega-7)^2 = 4$

$\omega = 7 + (\pm 2) = 5$ or 9

$\Rightarrow P[Z=4] = P(\{5\}) + P(\{9\}) = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$

b. Find $P[Z>20]$.

Method 1:

$Z(\omega) > 20$ when $(\omega-7)^2 > 20$

$\cancel{\omega > \underbrace{7+\sqrt{20}}_{11.4721}}$ or $\omega < \underbrace{7-\sqrt{20}}_{\approx 2.5279}$

none of the ω in Ω satisfies this condition

$\omega = 0, 1, 2$

$\Rightarrow P[Z>20] = P(\{0\}) + P(\{1\}) + P(\{2\}) = \frac{3}{10}$

Method 2: Because Ω is not large, it is possible to find $X(\omega)$ for all ω.

ω	$\omega - 7$	$(\omega-7)^2$
0	−7	49
1	−6	36
2	−5	25
3	−4	16
4	−3	9
5	−2	4
6	−1	1
7	0	0
8	1	1
9	2	4

>20

same

ECS 315: In-Class Exercise # 11

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **16** / **10** / 2018

Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. Consider a random experiment in which you roll a six-sided fair dice (whose faces are numbered 1-6). We define a random variable *X* by:

$$X(\omega)=(\omega-3)(\omega-5).$$

a. Find all possible values of the random variable *X*.

ω	$\omega-3$	$\omega-5$	$X(\omega)$
1	-2	-4	8
2	-1	-3	3
3	0	-2	0
4	1	-1	-1
5	2	0	0
6	3	1	3

The possible values of X are

$$\underbrace{-1, 0, 3, 8}$$

Remark: This forms the "default" support of X.

b. Find its probability mass function $p_X(x)$. $\equiv P[X=x]$

From part (a), we know that $p_X(x)=0$ when $x \notin \{-1, 0, 3, 8\}$.

So we only need to find $p_X(x)$ when $x = -1, 0, 3, 8$

$p_X(-1) \equiv P[X=-1] = P(\{4\}) = 1/6$

$p_X(0) \equiv P[X=0] = P(\{3,5\}) = 2/6 = 1/3$

$p_X(3) \equiv P[X=3] = P(\{2,6\}) = 2/6 = 1/3$

$p_X(8) \equiv P[X=8] = P(\{1\}) = 1/6$

$$p_X(x) = \begin{cases} 1/6, & x = -1, 8 \\ 1/3, & x = 0, 3 \\ 0, & \text{otherwise} \end{cases}$$

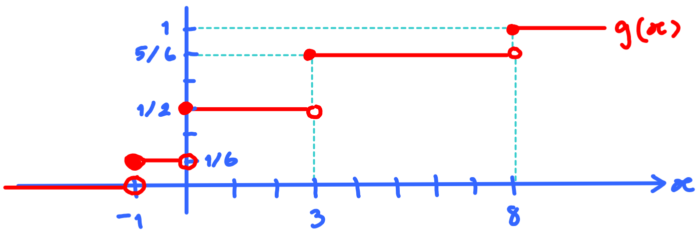
c. $P[X \le 1]$

X can be -1, 0, 3, 8.

Among these values, those that are "≤1" are -1 and 0.

Therefore, $P[X \le 1] = p_X(-1) + p_X(0) = \frac{1}{6} + \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$

d. (optional) Plot the function $g(x) = P[X \le x]$.



ECS 315: In-Class Exercise # 12

Instructions

1. Separate into groups of no more than three persons. The group cannot be the same as any of your former groups after the midterm.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **18** / **10** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. Consider a random variable X whose pmf is given by

$$p_X(x)=\begin{cases}0.4, & x=1,\\ 0.3, & x=2,3,\\ 0, & \text{otherwise.}\end{cases}$$

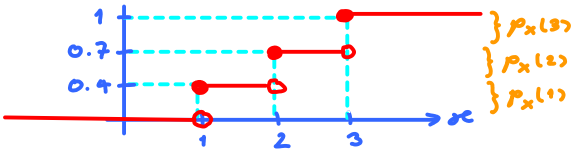
a. Find $P\left[X\le \overset{\approx 1.414}{\sqrt{2}}\right]$.

The possible values of X are 1, 2, and 3. Among these, only "1" is "$\le\sqrt{2}$".

Therefore, $P[X\le\sqrt{2}] = p_X(1) = 0.4$

b. Plot the cdf of this random variable.

Recall that the cdf can be derived from the pmf by using the $p_X(x)$ as the jump amount at x.



2. Consider a random variable X whose cdf is given by

$$F_X(x)=\begin{cases}0, & x<0,\\ 0.3, & 0\le x<2,\\ 1, & x\ge 2.\end{cases}$$

a. Find $P[X\le 1]$.

By definition, $F_X(x) \equiv P[X\le x]$.

Therefore, $P[X\le 1] \equiv F_X(1) = 0.3$.

b. Find $P[X>1]$.

Because "$X>1$" is the opposite of "$X\le 1$",

we know that $P[X>1] = 1-P[X\le 1] = 1-0.3 = 0.7$

c. Plot the pmf of X.

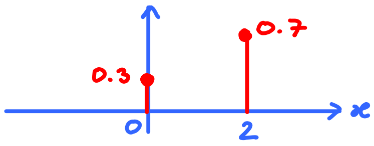
For discrete RV, the pmf can be derived from the jump amounts in the cdf.

Here, the jumps in the cdf happen two times: at $x=0$ and at $x=2$.

The jump amounts are 0.3 and 0.7, respectively.

Therefore, $p_X(x)=\begin{cases}0.3, & x=0,\\ 0.7, & x=2,\\ 0, & \text{otherwise.}\end{cases}$

Note that we always use stem plot for pmf.



ECS 315: In-Class Exercise # 13

Date: **25** / **10** / 2018

Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Consider the random variable specified in each part below.

i) Write down its (minimal) support.

ii) Find $P[X=1]$. Your answer should be of the form 0.XXXX.

iii) Find $P[X=4]$. Your answer should be of the form 0.XXXX.

	S_X (minimal) support	$P[X=1] = p_X(1)$	$P[X=4] = p_X(4)$
$X \sim \text{Bernoulli}\left(\overset{p}{\tfrac{2}{3}}\right)$	$\{0,1\}$	$= p = \frac{2}{3} \approx 0.6667$	0.0000 because "4" is not in the support
binomial $X \sim \mathcal{B}\left(3, \frac{2}{3}\right)$ n p	$\{0,1,2,3\}$	$= \binom{n}{1} p^1 (1-p)^{n-1}$ $= \binom{3}{1} \frac{2}{3} \left(\frac{1}{3}\right)^2$ $= \cancel{3} \times \frac{2}{\cancel{3}} \times \frac{1}{9} = \frac{2}{9} \approx 0.2222$	0.0000 because "4" is not in the support
uniform $X \sim \mathcal{U}(\{2,3,4\})$	$\{2,3,4\}$	0.0000 because "1" is not in the support	$\lvert\{2,3,4\}\rvert = 3$ So, $P[X=4] = \frac{1}{3} \approx 0.3333$
geometric $X \sim \mathcal{G}_1\left(\frac{2}{3}\right)$ p	$\{1,2,3,4,\ldots\}$	$= p(1-p)^{1-1}$ $= \frac{2}{3} \times 1 = \frac{2}{3} \approx 0.6667$	$= p(1-p)^{4-1}$ $= \frac{2}{3} \times \left(\frac{1}{3}\right)^3 = \frac{2}{3^4} = \frac{2}{81}$ ≈ 0.0247
Poisson $X \sim \mathcal{P}(3)$ α	$\{0,1,2,3 \ldots\}$	$= e^{-3} \frac{3^1}{1!} = 3e^{-3}$ ≈ 0.1494	$= e^{-3} \frac{3^4}{4!} = e^{-3} \frac{3 \times 3 \times 3 \times \cancel{3}}{4 \times \cancel{3} \times 2 \times 1}$ $= \frac{27}{8} e^{-3} \approx 0.1680$

ECS 315: In-Class Exercise # 14

Date: **30** / **10** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. Suppose $X \sim \mathcal{G}_0\left(\frac{1}{2}\right)$. Plot its cdf $F_X(x)$ on the interval $[-3,3]$.

$S_X = \{0, 1, 2, \ldots\}$

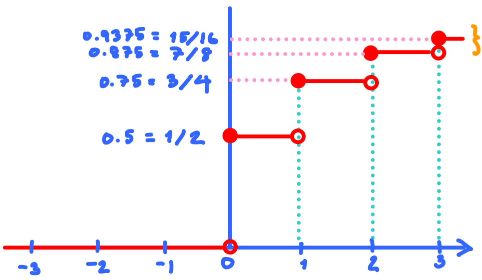
$$p_X(x) = \begin{cases} p(1-p)^x, & x = 0, 1, 2, \ldots, \\ 0, & \text{otherwise.} \end{cases}$$

$P[X=0] = \frac{1}{2}$

$P[X=1] = \frac{1}{4}$

$P[X=2] = 1/8$

$P[X=3] = 1/16$



}After this, the graph still increases in even smaller steps. However, we stop here because we are asked to focus on on the interval [-3,3].

2. Arrivals of customers at a local supermarket are modeled by a Poisson process with a rate of $\lambda = 0.5$ customers per minute. Let M be the number of customers arriving between 10:54 AM and 11:00 AM. What is the probability that $M \geq 2$?

$$\left.\begin{aligned} \lambda &= 0.5 \\ \tau &= 6 \end{aligned}\right\} \alpha = \lambda\tau = 0.5 \times 6 = 3$$

$$p_M(m) = e^{-\alpha}\frac{\alpha^m}{m!}$$

$$P[M<2] = \overset{0.0498}{P[M=0]} + \overset{0.1494}{P[M=1]}$$

$$= e^{-\alpha} + \alpha e^{-\alpha} = (\alpha+1)e^{-\alpha} \approx 0.1991$$

$$P[M \geq 2] = 1 - (\alpha+1)e^{-\alpha} = 1 - 4e^{-3} \approx 0.8009$$

3. Consider (a sequence of independent) Bernoulli trials whose success probability for each trial is $1/5$. For each of the random variables defined below, indicate the **name and the parameter(s)** of the family it belongs to.

Random Variable	Family
may fail 0 time. K = the number of failures until the first success occurs.	$\mathcal{G}_0(1/5)$
N = the number of successes among the first 7 trials.	$\mathcal{B}(7, 1/5)$
(Optional) Suppose we know that [there is exactly one success during the first 7 trials] ← A. Let M = the trial position in which that success occurs	$\mathcal{U}(\{1,2,\ldots,7\})$

This suggests conditional probability

Let B_k be the event that the results of the Bernoulli trials are F..F S F..F

failure ↗ ← success

↑ k^{th} position

$$P[M=k|A] = P(B_k|A) = \frac{P(B_k \cap A)}{P(A)} = \frac{\cancel{p^1(1-p)^6}}{\binom{7}{1}\cancel{p^1(1-p)^6}}$$

ECS 315: In-Class Exercise # 15

Date: **01** / **11** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. You are given an unfair coin with probability of obtaining a heads equal to $\overset{p}{2\times10^{-17}}$ You toss this coin $\underset{n}{2.5\times10^{16}}$ times. Use **<u>Poisson approximation</u>** to find the probability that you get "tails for all the tosses".

Let N be this RV

The $\overbrace{\text{number of successes}}$ (Hs) in n Bernoulli trials is binomial(n,p) where p is the success probability for each trial.

Here we want to find $P[N=0]$

When n is large and p is small, the binomial RV can be approxinated by a Poisson(α) RV where $\alpha = np$

2.5×10^{16} is large $\quad$ 2×10^{-17} is small

Here, $\alpha = n\times p = 2.5\times10^{16}\times2\times10^{-17} = 5\times10^{-1} = 0.5 \Rightarrow P[N=0] \approx e^{-\alpha}\frac{\alpha^0}{0!} = e^{-\alpha} = e^{-0.5} \approx 0.6065$

2. Find the expected value of the random variable X defined in each part below:

a. $p_X(x) = \begin{cases} \frac{x+2}{8}, & x \in \{-1,1,2\}, \\ 0, & \text{otherwise.} \end{cases}$

x	$p_X(x)$
-1	$\frac{-1+2}{8} = \frac{1}{8}$
1	$\frac{1+2}{8} = \frac{3}{8}$
2	$\frac{2+2}{8} = \frac{4}{8}$

Check: $\frac{1}{8}+\frac{3}{8}+\frac{4}{8} = 1$ ✓

$\mathbb{E}X = \sum_x x\,p_X(x) = (-1)\frac{1}{8} + (1)\frac{3}{8} + (2)\frac{4}{8} = \frac{10}{8} = \frac{5}{4} = 1.25$

b. $p_X(x) = \begin{cases} 0.25, & x = 1,3, \\ c, & x = 2, \\ 0, & \text{otherwise.} \end{cases}$

x	$p_X(x)$
1	0.25
2	$c = 0.5$
3	0.25

"$\Sigma = 1$"

$0.25 + c + 0.25 = c + 0.5 = 1$

$c = 0.5$

$\mathbb{E}X = \sum_x x\,p_X(x) = (1)0.25 + (2)0.5 + (3)0.25 = 0.25 + 1 + 0.75 = 2$

c. $F_X(x) = \begin{cases} 0, & x < 0, \\ 0.3, & 0 \le x < 2, \\ 1, & x \ge 2. \end{cases}$

This cdf has two jumps; one is @ $x=0$ and another one is @ $x=1$.

The jump sizes are 0.3 and $1-0.3=0.7$, respectively.

$\Rightarrow p_X(x) = \begin{cases} 0.3, & x=0, \\ 0.7, & x=2, \\ 0, & \text{otherwise.} \end{cases}$

$\mathbb{E}X = \sum_x x\,p_X(x) = (0)0.3 + (2)0.7 = 1.4$

ECS 315: In-Class Exercise # 16

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **06** / **11** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Find $\mathbb{E}[X^2]$, $\mathbb{E}[(X+1)^2]$, and $\mathrm{Var}[X]$ of the random variable X defined below:

x | $p_X(x)$ -1 | $\frac{-1+2}{8} = \frac{1}{8}$ 1 | $\frac{1+2}{8} = \frac{3}{8}$ 2 | $\frac{2+2}{8} = \frac{4}{8}$	$p_X(x) = \begin{cases} \frac{x+2}{8}, & x \in \{-1,1,2\}, \\ 0, & \text{otherwise.} \end{cases}$	$p_X(x) = \begin{cases} 0.25, & x = 1,3, \\ 0.5, & x = 2, \\ 0, & \text{otherwise.} \end{cases}$
$\mathbb{E}[X]$	1.25	2
$\mathbb{E}[X^2]$ $= \sum_x x^2 p_X(x)$	$= (-1)^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{4}{8}$ $= \frac{1}{8}(1+3+16) = \frac{20}{8} = \frac{5}{2} = 2.5$	$= 1^2 \times \frac{1}{4} + 2^2 \times \frac{1}{2} + 3^2 \times \frac{1}{4}$ $= \frac{1}{4} + 2 + \frac{9}{4} = 2 + \frac{10}{4} = 2 + \frac{5}{2} = 4.5$
$\mathbb{E}[(X+1)^2]$ $= \sum_x (x+1)^2 p_X(x)$	$= (-1+1)^2 \times \frac{1}{8} + (1+1)^2 \times \frac{3}{8} + (2+1)^2 \times \frac{4}{8}$ $= 0 + \frac{4\times3+9\times4}{8} = \frac{12}{2} = 6$ Alternatively, $\mathbb{E}[(X+1)^2] = \mathbb{E}[X^2+2X+1]$ $= \mathbb{E}[X^2] + 2\mathbb{E}X + 1 = 2.5 + 2\times\frac{5}{4} + 1 = 6$	$= (1+1)^2 \times \frac{1}{4} + (2+1)^2 \times \frac{1}{2} + (3+1)^2 \times \frac{1}{4}$ $= \frac{4}{4} + \frac{9}{2} + \frac{16}{4} = 1 + 4.5 + 4 = 9.5$ Alternatively, $\mathbb{E}[(X+1)^2] = \mathbb{E}[X^2] + 2\mathbb{E}X + 1$ $= 4.5 + 2\times2 + 1 = 9.5$
$\mathrm{Var}[X]$	$= \mathbb{E}[X^2] - (\mathbb{E}X)^2 = \frac{5}{2} - \left(\frac{5}{4}\right)^2$ $= \frac{5}{2} - \frac{25}{16} = \frac{40-25}{16} = \frac{15}{16} = 0.9375$	$= \mathbb{E}[X^2] - (\mathbb{E}X)^2 = 4.5 - 2^2$ $= 4.5 - 4 = 0.5$

ECS 315: In-Class Exercise # 17

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **13** / **11** / 2018

Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

In this question, we consider two distributions for a random variable X. In part (a), which corresponds to the second column in the table below, X is a ***discrete*** random variable with its pmf specified in the first row. In part (b), which corresponds to the third column, X is a ***continuous*** random variable with its pdf specified in the first row.

	$p_X(x) = \begin{cases} cx^2, & x \in \{1,3\}, \\ 0, & \text{otherwise.} \end{cases}$	$f_X(x) = \begin{cases} cx^2, & x \in (1,3), \\ 0, & \text{otherwise.} \end{cases}$
Find *c*	"$\sum = 1$" : $p_X(1) + p_X(3) = 1$ $c\,1^2 + c\,3^2 = 1$ $10c = 1$ $c = \frac{1}{10}$	$\int_{-\infty}^{\infty} f_X(x)dx = \int_1^3 cx^2 dx = c\frac{x^3}{3}\Big\|_1^3$ $= 9c - \frac{c}{3} = c\frac{26}{3}$ "$\int = 1$" : $c\frac{26}{3} = 1 \Rightarrow c = \frac{3}{26} \approx 0.1154$
Find $P[X=1]$	$P[X=1] = p_X(1) = cx^2\Big\|_{x=1} = c = \frac{1}{10}$	Method 1: X is a continuous RV. $P[X=x] = 0$ for any x. Therefore, $P[X=1] = 0$. Method 2: X is a continuous RV. $P[X=1] = \int_1^1 f_X(x)dx = \int_1^1 cx^2 dx = c\frac{x^3}{3}\Big\|_1^1$ $= 0$
Find $P[1<X<2]$	The possible values of this RV are 1 and 3. So, there is no possible value in the open interval (1,2). Therefore, $P[1<X<2] = 0$	$P[1<X<2] = \int_1^2 f_X(x)dx = \int_1^2 cx^2 dx$ $= c\frac{x^3}{3}\Big\|_1^2 = \frac{c}{3}(2^3 - 1^3) = \frac{7c}{3}$ $= \frac{7}{3} \times \frac{3}{26} = \frac{7}{26} \approx 0.2692$
Find $P[X>2]$	Again, the possible values of this RV are 1 and 3. Only "3" satisfies the condition ">2". Therefore, $P[X>2] = p_X(3) = cx^2\Big\|_{x=3} = 9c = \frac{9}{10}$	Method 1: $P[X>2] = \int_2^{\infty} f_X(x)dx = \int_2^3 cx^2 dx$ $= c\frac{x^3}{3} = \frac{c}{3}(3^3 - 2^3) = c\frac{19}{3}$ $= \frac{3}{26} \times \frac{19}{3} = \frac{19}{26} \approx 0.7308$ Method 2: Consider the following partition of Ω: $\Omega = [X \le 1] \cup [1<X<2] \cup [X=2] \cup [X>2]$

By finite additivity, we have

$P(\Omega) = P[X \le 1] + P[1<X<2] + P[X=2] + P[X>2]$

$1 = 0 + \frac{7}{26} + 0 + P[X>2]$

$\Rightarrow P[X>2] = 1 - \frac{7}{26} = \frac{19}{26}$

ECS 315: In-Class Exercise # 18

Instructions

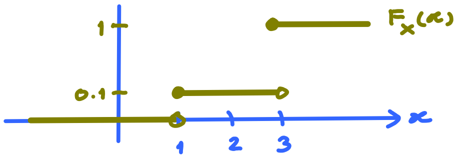
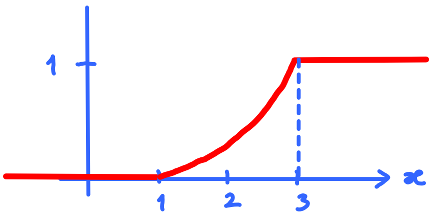
1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **15** / **11** / 2018

Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

1. In this question, we consider two distributions for a random variable X. In part (a), which corresponds to the second column in the table below, X is a ***discrete*** random variable with its pmf specified in the first row. In part (b), which corresponds to the third column, X is a ***continuous*** random variable with its pdf specified in the first row.

$p_X(1) = 0.1$ $p_X(3) = 0.9$

	$p_X(x) = \begin{cases} \frac{1}{10}x^2, & x \in \{1,3\}, \\ 0, & \text{otherwise.} \end{cases}$	$f_X(x) = \begin{cases} \frac{3}{26}x^2, & x \in (1,3), \\ 0, & \text{otherwise.} \end{cases}$
Find the cdf $F_X(x)$	For discrete RV, it may be easier to work on the plot of cdf first, then come back here. $F_X(x) = \begin{cases} 0, & x < 1, \\ 0.1, & 1 \le x < 3, \\ 1, & x \ge 3. \end{cases}$	For $x \in (1,3)$, $F_X(x) = \int_1^x f_X(t)\,dt = \int_1^x \frac{3}{26}t^2\,dt = \frac{3}{26}\left.\frac{t^3}{3}\right\|_1^x$ $= \frac{1}{26}(x^3-1)$ $F_X(x) = \begin{cases} 0, & x \le 1, \\ \frac{1}{26}(x^3-1), & 1 < x < 3, \\ 1, & \text{otherwise} \end{cases}$
Plot the cdf $F_X(x)$		
Find $\mathbb{E}X$	$\mathbb{E}X = \sum_x x\,p_X(x)$ $= 1\times 0.1 + 3\times 0.9$ $= 0.1 + 2.7$ $= 2.8$	$\mathbb{E}X = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_1^3 x \frac{3}{26}x^2\,dx$ $= \frac{3}{26}\left.\frac{x^4}{4}\right\|_1^3 = \frac{3}{4\times 26}(3^4-1^4)$ $= \frac{3}{\cancel{4}\times\underset{13}{\cancel{26}}} \times \overset{10\ \cancel{20}}{\cancel{80}} = \frac{30}{13} \approx 2.3077$
Find $\mathbb{E}\left[X^2\right]$	$\mathbb{E}[X^2] = \sum_x x^2 p_X(x)$ $= 1^2\times 0.1 + 3^2\times 0.9$ $= 0.1 + 8.1$ $= 8.2$	$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_1^3 x^2 \frac{3}{26}x^2\,dx$ $= \frac{3}{26}\left.\frac{x^5}{5}\right\|_1^3 = \frac{3}{26\times 5}(3^5-1^5)$ $= \frac{3}{\underset{13}{\cancel{26}}\times 5} \times \overset{121}{\cancel{242}} = \frac{363}{65} \approx 5.58$

ECS 315: In-Class Exercise # 19

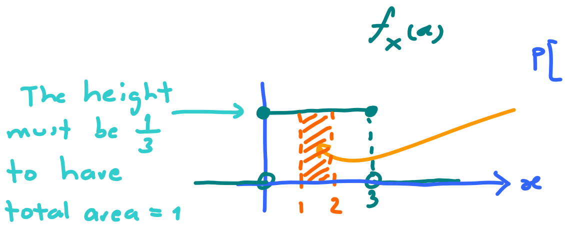
Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **22** / **11** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Calculate $P[1 < X \leq 2]$ for each of the following random variables. Your answer should be of the form 0.XXXX.

a) $X \sim \mathcal{U}(0,3)$



$P[1 \leq X \leq 2] = 1 \times \frac{1}{3} = \frac{1}{3} \approx 0.3333$

Alternatively, the cdf of $\mathcal{U}(0,3)$ is

$$F_X(x) = \begin{cases} \frac{x-0}{3-0}, & 0 \leq x \leq 3, \\ 0, & \text{otherwise} \end{cases}$$

Therefore, $P[1 \leq X \leq 2] = F_X(2) - F_X(1) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

b) $X \sim \mathcal{E}(3)$ $\uparrow\!\!\_\, \lambda$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases} = \begin{cases} 3e^{-3x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$P[1 \leq X \leq 2] = \int_1^2 f_X(x)\,dx = \int_1^2 3e^{-3x}\,dx = 3\frac{e^{-3x}}{-3}\Big|_1^2 = e^{-3} - e^{-6} \approx 0.0473$$

Alternatively, the cdf of $\mathcal{E}(3)$ is

$$F_X(x) = \begin{cases} 1 - e^{-3x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Therefore, $P[1 \leq X \leq 2] = F_X(2) - F_X(1) = (1 - e^{-6}) - (1 - e^{-3}) = e^{-3} - e^{-6}$

c) $X \sim \mathcal{N}(0,1)$

$= \Phi(2) - \Phi(1) \approx 0.97725 - 0.8413 \approx 0.13595$

d) $X \sim \mathcal{N}(1,3)$

$= \Phi\left(\frac{2-1}{\sqrt{3}}\right) - \Phi\left(\frac{1-1}{\sqrt{3}}\right) = \Phi\left(\frac{1}{\sqrt{3}}\right) - \Phi(0) \approx \Phi(0.58) - 0.5$

$\approx 0.7190 - 0.5 = 0.2190$

ECS 315: In-Class Exercise # 20

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **29** / **11** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

[F2013] Random variables X and Y have the following joint pmf

$$p_{X,Y}(x,y)=\begin{cases} c(x+y), & x\in\{1,3\} \text{ and } y\in\{1,3\}, \\ 0, & \text{otherwise.}\end{cases}$$

a) Find c

$$x+y \qquad \begin{array}{c|cc} x\backslash y & 1 & 3 \\ \hline 1 & 2 & 4 \\ 3 & 4 & 6 \end{array}$$

For a joint pmf, "$\Sigma = 1$"

$$\Rightarrow 2c+4c+4c+6c=1$$

$$16c = 1$$

$$c = 1/16$$

b) Find the joint pmf matrix $\mathbf{P}_{X,Y}$

$$\mathbf{P}_{X,Y} = \begin{array}{c|cc} x\backslash y & 1 & 3 \\ \hline 1 & 2/16 & 4/16 \\ 3 & 4/16 & 6/16 \end{array} = \begin{array}{c|cc} x\backslash y & 1 & 3 \\ \hline 1 & 1/8 & 1/4 \\ 3 & \boxed{1/4} & 3/8 \end{array}$$

c) Find $P[X-Y>1]$

$$x-y \qquad \begin{array}{c|cc} x\backslash y & 1 & 3 \\ \hline 1 & 0 & -2 \\ 3 & \boxed{2} & 0 \end{array}$$

only this position satisfies the condition ">1".

$$P[X-Y>1] = \frac{1}{4}$$

d) Find the pmf $p_X(x)$ and the pmf $p_Y(y)$.

$$\mathbf{P}_{X,Y} = \begin{array}{c|cc|l} x\backslash y & 1 & 3 & \\ \hline 1 & 2/16 & 4/16 & \xrightarrow{\Sigma} 6/16 = 3/8 \\ 3 & 4/16 & 6/16 & \xrightarrow{\Sigma} 10/16 = 5/8 \\ \hline & \Sigma\downarrow & \Sigma\downarrow & \\ & 6/16 & 10/16 & \\ & = 3/8 & = 5/8 & \end{array}$$

$$p_X(x) = \begin{cases} 3/8, & x=1, \\ 5/8, & x=3, \\ 0, & \text{otherwise.}\end{cases}$$

$$p_Y(y) = \begin{cases} 3/8, & y=1, \\ 5/8, & y=3, \\ 0, & \text{otherwise.}\end{cases}$$

Note that X and Y are identically distributed.

e) Find $\text{Cov}[X,Y]$.

$$= \mathbb{E}[XY] - \mathbb{E}X\,\mathbb{E}Y = 5 - \left(\frac{9}{4}\right)^2 = \frac{80-81}{16} = -\frac{1}{16}$$

$$\mathbb{E}X = \sum_x x\,p_X(x) = 1\times\frac{3}{8} + 3\times\frac{5}{8} = \frac{3+15}{8} = \frac{18}{8} = \boxed{\frac{9}{4}} = \mathbb{E}Y$$

$$xy\,p_{X,Y}(x,y) \qquad \begin{array}{c|cc} x\backslash y & 1 & 3 \\ \hline 1 & 1\times\frac{1}{8} & 3\times\frac{2}{8} \\ 3 & 3\times\frac{2}{8} & 9\times\frac{3}{8} \end{array} \Rightarrow \mathbb{E}[XY] = \sum_{(x,y)} xy\,p_{X,Y}(x,y) = \frac{1+6+6+27}{8} = \frac{40}{8} = \boxed{5}$$