

ECS 315: Probability and Random Processes

2017/1

HW 5 — Due: Sep 28, 4 PM

*Lecturer: Prapun Suksompong, Ph.D.***Instructions**

- (a) This assignment has 4 pages.
- (b) (1 pt) Work and write your answers **directly on this sheet** (not on another blank sheet of paper). Hard-copies are distributed in class.
- (c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.
- (d) (8 pt) Try to solve all non-optional problems.
- (e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

**Problem 1.** Series Circuit: The circuit in Figure 5.1 operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates? [Montgomery and Runger, 2010, Ex. 2-32]



Figure 5.1: Circuit for Problem 1

**Problem 2.** In an experiment,  $A$ ,  $B$ ,  $C$ , and  $D$  are events with probabilities  $P(A \cup B) = \frac{5}{8}$ ,  $P(A) = \frac{3}{8}$ ,  $P(C \cap D) = \frac{1}{3}$ , and  $P(C) = \frac{1}{2}$ . Furthermore,  $A$  and  $B$  are disjoint, while  $C$  and  $D$  are independent.

(a) Find

(i)  $P(A \cap B)$

(ii)  $P(B)$

(iii)  $P(A \cap B^c)$

(iv)  $P(A \cup B^c)$

(b) Are  $A$  and  $B$  independent?

(c) Find

(i)  $P(D)$

(ii)  $P(C \cap D^c)$

(iii)  $P(C^c \cap D^c)$

(iv)  $P(C|D)$

(v)  $P(C \cup D)$

(vi)  $P(C \cup D^c)$

(d) Are  $C$  and  $D^c$  independent?

**Problem 3.** You have two coins, a fair one with probability of heads  $\frac{1}{2}$  and an unfair one with probability of heads  $\frac{1}{3}$ , but otherwise identical. A coin is selected at random and tossed, falling heads up. How likely is it that it is the fair one? [Capinski and Zastawniak, 2003, Q7.28]

**Problem 4.** You have three coins in your pocket, two fair ones but the third biased with probability of heads  $p$  and tails  $1-p$ . One coin selected at random drops to the floor, landing heads up. How likely is it that it is one of the fair coins? [Capinski and Zastawniak, 2003, Q7.29]

## Extra Questions

Here are some optional questions for those who want more practice.

**Problem 5.** In his book *Chances: Risk and Odds in Everyday Life*, James Burke says that there is a 72% chance a polygraph test (lie detector test) will catch a person who is, in fact, lying. Furthermore, there is approximately a 7% chance that the polygraph will falsely accuse someone of lying. [Brase and Brase, 2011, Q4.2.26]

- (a) If the polygraph indicated that 30% of the questions were answered with lies, what would you estimate for the actual percentage of lies in the answers?

- (b) If the polygraph indicated that 70% of the questions were answered with lies, what would you estimate for the actual percentage of lies?

**Problem 6.** Software to detect fraud in consumer phone cards tracks the number of metropolitan areas where calls originate each day. It is found that 1% of the legitimate users originate calls from two or more metropolitan areas in a single day. However, 30% of fraudulent users originate calls from two or more metropolitan areas in a single day. The proportion of fraudulent users is 0.01%. If the same user originates calls from two or more metropolitan areas in a single day, what is the probability that the user is fraudulent? [Montgomery and Runger, 2010, Q2-144]