Name ________________________ ID3 _ _ _

ECS 315: Probability and Random Processes **2017/1**

HW 3 — Due: Sep 14, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 4 pages.

(b) (1 pt) Work and write your answers **directly on this sheet** (not on another blank sheet of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. If A, B, and C are disjoint events with $P(A) = 0.2$, $P(B) = 0.3$ and $P(C) = 0.4$, determine the following probabilities:

(a) $P(A \cup B \cup C)$

(b) $P(A \cap B \cap C)$

(c) $P(A \cap B)$

(d) $P((A \cup B) \cap C)$

(e) $P(A^c \cap B^c \cap C^c)$

[Montgomery and Runger, 2010, Q2-75]

Problem 2. The sample space of a random experiment is $\{a, b, c, d, e\}$ with probabilities 0.1, 0.1, 0.2, 0.4, and 0.2, respectively. Let A denote the event $\{a, b, c\}$, and let B denote the event $\{c, d, e\}$. Determine the following:

(a) $P(A)$

(b) $P(B)$

(c) $P(A^c)$

(d) $P(A \cup B)$

(e) $P(A \cap B)$

[Montgomery and Runger, 2010, Q2-55]

Problem 3. ***Binomial theorem***: For any positive integer n, we know that

$$(x + y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}. \tag{3.1}$$

(a) What is the coefficient of $x^{12}y^{13}$ in the expansion of $(x + y)^{25}$?

(b) What is the coefficient of $x^{12}y^{13}$ in the expansion of $(2x - 3y)^{25}$?

(c) Use the binomial theorem (3.2) to evaluate $\sum\limits_{k=0}^{n} (-1)^k \binom{n}{k}$.

Problem 4. Let A and B be events for which $P(A)$, $P(B)$, and $P(A \cup B)$ are known. Express the following probabilities in terms of the three known probabilities above.

(a) $P(A \cap B)$

(b) $P(A \cap B^c)$

(c) $P\left(B \cup \left(A \cap B^c\right)\right)$

(d) $P(A^c \cap B^c)$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 5. ***Binomial theorem***: For any positive integer n, we know that

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}. \tag{3.2}$$

(a) Use the binomial theorem (3.2) to simplify the following sums

(i) $\sum\limits_{\substack{r=0 \\ r \text{ even}}}^{n} \binom{n}{r} x^r (1-x)^{n-r}$

(ii) $\sum\limits_{\substack{r=0 \\ r \text{ odd}}}^{n} \binom{n}{r} x^r (1-x)^{n-r}$

(b) If we differentiate (3.2) with respect to x and then multiply by x, we have

$$\sum_{r=0}^{n} r\binom{n}{r}x^r y^{n-r} = nx(x+y)^{n-1}.$$

Use similar technique to simplify the sum $\sum_{r=0}^{n} r^2\binom{n}{r}x^r y^{n-r}$.

Problem 6.

(a) Suppose that $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$. Find the range of possible values for $P(A \cap B)$. Hint: Smaller than the interval $[0, 1]$. [Capinski and Zastawniak, 2003, Q4.21]

(b) Suppose that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. Find the range of possible values for $P(A \cup B)$. Hint: Smaller than the interval $[0, 1]$. [Capinski and Zastawniak, 2003, Q4.22]