# ECS 315: In-Class Exercise # 5 **Solution**

## Instructions

1. Separate into groups of no more than three persons.
2. **The group cannot be the same as your former group.**
3. Only one submission is needed for each group.
4. **Do not panic.**

| Date: 12 / 09 / 2017 | | | |
|---|---|---|---|
| Name | ID (last 3 digits) | | |
| Prapun | 5 | 5 | 5 |
| | | | |
| | | | |

Consider the following sequences of 1s and 0s which summarize the data obtained from 16 testees in a disease testing experiment.

```
D:  0 1 1 0 0 0 0 1 1 1 1 0 1 0 1 1
TP: 0 0 0 0 1 0 1 1 0 0 1 1 0 0 1 1
```

The results in the *i*-th column are for the *i*-th testee. The D row indicates whether each of the testees actually has the disease under investigation. The TP row indicates whether each of the testees is tested positive for the disease.

Numbers "1" and "0" correspond to "True" and "False", respectively.

Suppose we randomly pick a testee from this pool of 16 persons. Let $D$ be the event that this selected person actually has the disease. Let $T_P$ be the event that this selected person is tested positive for the disease.

Find the following probabilities. No explanation is needed here.

| | |
|---|---|
| $P(D) =$ $\frac{9}{16}$ Among the 16 testees, 9 have the disease. | $P(T_P) =$ $\frac{7}{16}$ Among the 16 testees, 7 test positive. |
| $P(T_P \cap D) =$ $\frac{4}{16} = \frac{1}{4}$ Among the 16 testees, 4 have the disease and test positive. | $P(T_P \cap D^c) =$ $\frac{3}{16}$ Among the 16 testees, 3 test positive but do not have the disease. |
| $P(T_P \mid D) =$ $\frac{4}{9}$ Among the 9 testees who have the disease, 4 test positive. | $P(T_P \mid D^c) =$ $\frac{3}{16-9} = \frac{3}{7}$ Among the 16-9 = 7 testees who do not have the disease, 3 test positive. |
| $P(D \mid T_P) =$ $\frac{4}{7}$ Among the 7 testees who test positive, 4 have the disease. | $P(D \mid T_P^c) =$ $\frac{5}{16-7} = \frac{5}{9}$ Among the 16-7 = 9 testees who test negative, 5 have the disease. |
| $P(T_P^c \mid D) =$ $\frac{5}{9}$ Among the 9 testees who have the disease, 5 test negative. Alternatively, $= 1 - P(T_P \mid D) = 1 - \frac{4}{9} = \frac{5}{9}$ | $P(T_P \cup D) =$ $\frac{5+3+4}{16} = \frac{12}{16} = \frac{3}{4}$ $= P(T_P) + P(D) - P(T_P \cap D) = \frac{9+7-4}{16} = \frac{12}{16}$ ✓ |

check: $P(T_P) = P(T_P \mid D)P(D) + P(T_P \mid D^c)P(D^c) = \frac{4}{9} \times \frac{9}{16} + \frac{3}{7} \times \underbrace{\left(1 - \frac{9}{16}\right)}_{7/16} = \frac{7}{16}$ ✓