

ECS 315: In-Class Exercise 3 Solution

Sep 6, 2017

Instructions

1. Separate into groups of no more than three persons.
2. **The group cannot be the same as your former group.**
3. Only one submission is needed for each group.
4. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

| Name | ID |
|--------|-----|
| Prapun | 555 |
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1. Calculate the following quantities:

a. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

b. $\binom{10}{2} = \frac{10!}{8!2!} = \frac{10 \times 9}{2} = 5 \times 9 = 45$

c. $(10)_2 = 10 \times 9 = 90$

2. Suppose we sample 3 objects from a collection of 5 distinct objects.

Calculate the number of different possibilities when

a. the sampling is ordered and performed with replacement

$$n^r = 5 \times 5 \times 5 = 125$$

b. the sampling is ordered and performed without replacement

$$(n)_r = 5 \times 4 \times 3 = 60$$

c. the sampling is unordered and performed without replacement

$$\binom{n}{r} = \binom{5}{3} = \frac{5 \times 4}{2} = 10$$

3. Calculate the number of different results when we permute

a. ABCD

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

b. AAABBCC

$$\frac{7!}{3!2!2!} = \frac{7 \times 6 \times 5 \times 4}{2 \times 2} = 210$$

Extra part:

d. the sampling is unordered and performed with replacement

Note that we can't simply divide 125 by 3! to change from "ordered" to "unordered" case here because the sampling is done with replacement. Repeated uses of objects can potentially give groups with unequal sizes.

In class, we solve this by the bars-and-stars (or walls-ond-ones) argument.

Let n_i = the number of the i th object in the sample

Then, we must have

$$n_1 + n_2 + \dots + n_5 = 3.$$

The number of solution for the eqn. above is

$$\binom{3+4}{3} = \binom{7}{3} = \frac{7 \times 6 \times 5}{3!} = 35$$

(Note: The diagram shows a stars-and-bars representation with 3 stars and 4 walls, and a note ' wall' pointing to the walls.)*

Don't forget to simplify your answers.