

# ECS 315: In-Class Exercise #10 Solution

## Instructions

1. Separate into groups of no more than three persons. Only one submission is needed for each group. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: <u>07</u> / <u>11</u> /2017			
Name			ID (last 3 digits)
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Consider a random variable whose pmf is given by  $p_X(x) = \begin{cases} \frac{6}{11x}, & x=1,2,3, \\ 0, & \text{otherwise.} \end{cases}$

a) Find  $\mathbb{E}X = \sum_x x p_X(x) = \sum_x x \frac{6}{11x} = 3 \times \frac{6}{11} = \frac{18}{11}$

There are three values in the support of  $X$ . Therefore, the sum here has three  $x$ -values.

b) Let  $Y = (X - 2)^2$ .

a. Find  $p_Y(y)$ .

$p_X(x)$	$x$	$Y = (x-2)^2$
$\frac{6}{11 \times 1} = \frac{6}{11}$	1	1
$\frac{6}{11 \times 2} = \frac{3}{11}$	2	0
$\frac{6}{11 \times 3} = \frac{2}{11}$	3	1

$P[Y=0] = P[X=2] = \frac{6}{11 \times 2} = \frac{3}{11}$

$P[Y=1] = P[X=1] + P[X=3] = \frac{6}{11} + \frac{2}{11} = \frac{8}{11}$

$p_Y(y) = \begin{cases} 3/11, & y=0, \\ 8/11, & y=1, \\ 0, & \text{otherwise} \end{cases}$

b. Find  $\mathbb{E}Y = 0 \times \frac{3}{11} + 1 \times \frac{8}{11} = \frac{8}{11}$

Alternatively, with  $g(x) = (x-2)^2$ ,

$$\begin{aligned} \mathbb{E}[Y] &= \mathbb{E}[g(X)] = \sum_x g(x) p_X(x) = \sum_x (x-2)^2 \frac{6}{11x} \\ &= 1 \times \frac{6}{11} + 0 \times \frac{3}{11} + 1 \times \frac{2}{11} = \frac{8}{11} \end{aligned}$$