ECS 315: In-Class Exercise # 7 Solution

Date: 12 / 10 / 2017			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three persons. Only one submission is needed for each group. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Consider the outcome from a random experiment in which you roll a 10-sided fair dice. We define the following random variables from the outcomes of this experiment:

$$X(\omega) = \omega \quad \text{and} \quad Y(\omega) = (\omega - 7)^2.$$

(a) Find the sample space Ω for this experiment.

See next page if you start with "0".

$\Omega = \{1, 2, 3, \ldots, 10\}$

Note that because the dice is fair,

$P(\{\omega\}) = \frac{1}{|\Omega|} = \frac{1}{10}$ for any $\omega \in \Omega$.

(b) Find $P[X = 7]$.

Recall that we use square brackets to define an event from a statement about RV.

$[X = 7] = \{\omega \in \Omega : X(\omega) = 7\} = \{7\}$

$P[X = 7] = P([X = 7]) = P(\{7\}) = \frac{1}{10}$

(c) [M2016Q10]

Find $P[Y > 10]$.

Note that $Y(\omega) > 10 \equiv (\omega - 7)^2 > 10$

$\equiv \omega \in \{1, 2, 3\}$

Therefore, $[Y > 10] = \{1, 2, 3\}$

and

$P[Y > 10] = P(\{1, 2, 3\})$

$= P(\{1\}) + P(\{2\}) + P(\{3\})$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

$= \frac{3}{10} = 0.3.$

Because $|\Omega| = 10$, it is easy to simply test each value of ω by plugging-in to $(\omega - 7)^2$

ω	$\omega - 7$	$(\omega - 7)^2$
1	−6	36
2	−5	25
3	−4	16
4	−3	9
5	−2	4
6	−1	1
7	0	0
8	1	1
9	2	4
10	3	9

Alternatively, from $(\omega - 7)^2 > 10$, we must have

$\omega - 7 > \sqrt{10}$ or $\omega - 7 < -\sqrt{10}$

$\omega > 7 + \sqrt{10}$ | $\omega < 7 - \sqrt{10}$

$\omega > 10.1623$ | $\omega < 3.8377$

$\Downarrow$ none of the ω in Ω satisfies this | $\Downarrow$ $\omega = 1, 2, 3$

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3. **Do not panic.**

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Consider the outcome from a random experiment in which you roll a 10-sided fair dice. We define the following random variables from the outcomes of this experiment:

$$X(\omega) = \omega \quad \text{and} \quad Y(\omega) = (\omega - 7)^2.$$

(a) Find the sample space Ω for this experiment.

$\Omega = \{0, 1, 2, \ldots, 9\}$

Note that because the dice is fair, $P(\{\omega\}) = \frac{1}{|\Omega|} = \frac{1}{10}$ for any $\omega \in \Omega$.

(b) Find $P[X = 7]$.

Recall that we use square brackets to define an event from a statement about RV.

$[X = 7] = \{\omega \in \Omega : X(\omega) = 7\} = \{7\}$

$P[X = 7] = P([X = 7]) = P(\{7\}) = \frac{1}{10}$

(c) [M2016Q10]

Find $P[Y > 10]$.

Note that $Y(\omega) > 10 \equiv (\omega - 7)^2 > 10$

$\equiv \omega \in \{0, 1, 2, 3\}$

Because $|\Omega| = 10$, it is easy to simply test each value of ω by plugging-in to $(\omega - 7)^2$

ω	$\omega - 7$	$(\omega - 7)^2$
0	−7	49
1	−6	36
2	−5	25
3	−4	16
4	−3	9
5	−2	4
6	−1	1
7	0	0
8	1	1
9	2	4

Therefore, $[Y > 10] = \{0, 1, 2, 3\}$

and

$P[Y > 10] = P(\{0, 1, 2, 3\})$

$= P(\{1\}) + P(\{1\}) + P(\{2\}) + P(\{3\})$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

$= \frac{4}{10} = 0.4.$

Alternatively, from $(\omega - 7)^2 > 10$, we must have

$\omega - 7 > \sqrt{10}$ or $\omega - 7 < -\sqrt{10}$

$\omega > 7 + \sqrt{10}$ $\qquad$ $\omega < 7 - \sqrt{10}$

$\omega > 10.1623$ $\qquad$ $\omega < 3.8377$

$\Downarrow$ $\qquad$ $\Downarrow$

none of the ω in Ω satisfies this $\qquad$ $\omega = 0, 1, 2, 3$

ECS 315: In-Class Exercise # 8 Solution

Date: 19 / 10 / 2017			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three persons. Only one submission is needed for each group. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Consider a random variable whose pmf is given by $p_X(x) = \begin{cases} \frac{1}{4}, & x = 1, 9, \\ c, & x = 4, \\ 0, & \text{otherwise.} \end{cases}$

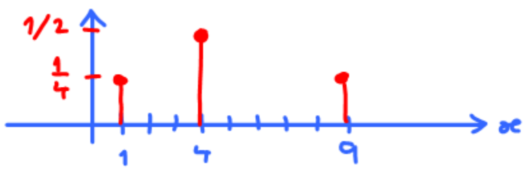
x	$p_X(x)$
1	1/4
4	c
9	1/4

a) Find the constant *c*.

$\sum_x p_X(x) = 1 \Rightarrow p_X(1) + p_X(4) + p_X(9) = 1$

$\frac{1}{4} + c + \frac{1}{4} = 1 \Rightarrow c = \frac{1}{2}$

b) Plot $p_X(x)$. (Recall that we use stem plot for pmf.)



c) Find $P[X \le 5]$.

$P[X \le 5] = P[X=4] + P[X=1] = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.

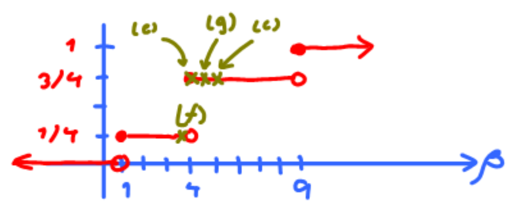
d) Find $P[X > 4]$. $= P[X=9] = \frac{1}{4}$

e) Find $P[X \le 4]$. $= P[X=1] + P[X=4] = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

f) Find $P[X \le 3.99]$. $= P[X=1] = \frac{1}{4}$

g) Find $P[X \le 4.01]$. $= P[X=1] + P[X=4] = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$.

(h) (Optional) Plot $P[X \le \beta]$



In section 8.2, this function of "β" is called the cumulative distribution function (cdf) of the RV X. There, we replace β by a lower-case x.

ECS 315: In-Class Exercise #9 **Solution**

Date: 02/ 11 / 2017			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Instructions

1. Separate into groups of no more than three persons. Only one submission is needed for each group. The group cannot be the same as any of your former groups.
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Consider the random variable specified in each part below.

i) Write down its (minimal) support.
ii) Write down its pmf.
iii) Find $P[X < 1]$
iv) Find $P[1 < X \le 2]$

The supports for all of these RVs contain 0, 1, ... Therefore, $P[X<1]=P[X=0]$

All of these RVs are integer-valued. Therefore, $P[1<X\le 2]=P[X=2]$

		Support	pmf	$P[X < 1]$	$P[1 < X \le 2]$
(a)	$X \sim \text{Bernoulli}\left(\frac{1}{2}\right)$	$\{0,1\}$	$\begin{cases} 1-p, & x=0, \\ p, & x=1, \\ 0, & \text{otherwise}. \end{cases} \overset{p=1/2}{=} \begin{cases} 1/2, & x=0,1, \\ 0, & \text{otherwise} \end{cases}$	$1/2$	$0.$
(b)	$X \sim \text{Binomial}\left(4,\frac{1}{4}\right)$ $n=4, p=1/4$	$\{0,1,2,3,4\}$	$\begin{cases} \binom{4}{x}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{4-x}, & x=0,1,2,3,4, \\ 0, & \text{otherwise} \end{cases} = \begin{cases} 81/256 \approx 0.3164, & x=0, \\ 27/64 \approx 0.4219, & x=1, \\ 27/128 \approx 0.2109, & x=2, \\ 3/64 \approx 0.0468, & x=3, \\ 1/256 \approx 0.0039, & x=4, \\ 0, & \text{otherwise} \end{cases}$	$\frac{81}{256} \approx 0.3164$	$\frac{27}{128} \approx 0.2109$
(c)	$X \sim \text{Poisson}(1)$ $\alpha = 1$	$\{0,1,2,\dots\}$	$\begin{cases} e^{-\alpha}\frac{\alpha^x}{x!}, & x=0,1,2,\dots \\ 0, & \text{otherwise} \end{cases} \overset{\alpha=1}{=} \begin{cases} \frac{1}{e x!}, & x=0,1,2,\dots, \\ 0, & \text{otherwise}. \end{cases}$	$\frac{1}{e} \approx 0.3679$	$\frac{1}{2e} \approx 0.1839$

ECS 315: In-Class Exercise #10 Solution

Date: 07 / 11 / 2017			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three persons. Only one submission is needed for each group. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Consider a random variable whose pmf is given by $p_X(x) = \begin{cases} \frac{6}{11x}, & x = 1,2,3, \\ 0, & \text{otherwise.} \end{cases}$.

a) Find $\mathbb{E}X$ $= \sum_x x\, p_X(x) = \sum_x \cancel{x} \frac{6}{11\cancel{x}} = 3 \times \frac{6}{11} = \frac{18}{11}$

There are three values in the support of X. Therefore, the sum here has three x-values.

b) Let $Y = (X-2)^2$.

a. Find $p_Y(y)$.

$p_X(x)$	x	$y=(x-2)^2$
$\frac{6}{11\times 1} = \frac{6}{11}$	1	1
$\frac{6}{11\times 2} = \frac{3}{11}$	2	0
$\frac{6}{11\times 3} = \frac{2}{11}$	3	1

$P[Y=0] = P[X=2] = \frac{6}{11\times 2} = \frac{3}{11}$

$P[Y=1] = P[X=1] + P[X=3] = 1 - \frac{3}{11} = \frac{8}{11}$

$p_Y(y) = \begin{cases} 3/11, & y=0, \\ 8/11, & y=1, \\ 0, & \text{otherwise} \end{cases}$

b. Find $\mathbb{E}Y$. $= 0 \times \frac{3}{11} + 1 \times \frac{8}{11} = \frac{8}{11}$

Alternatively, with $g(x) = (x-2)^2$,

$$\mathbb{E}[Y] = \mathbb{E}[\underbrace{g(X)}_{Y}] = \sum_x g(x)\, p_X(x) = \sum_x (x-2)^2 \frac{6}{11x}$$

$$= 1 \times \frac{6}{11} + 0 \times \frac{3}{11} + 1 \times \frac{2}{11} = \frac{8}{11}$$

ECS 315: In-Class Exercise #11 **Solution**

Instructions

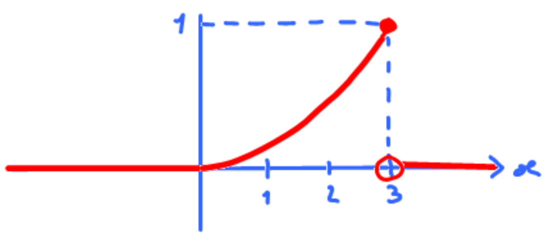
1. Separate into groups of no more than three persons. Only one submission is needed for each group. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **16** / **11** / 2017			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Consider a continuous random variable whose pdf is given by $f_X(x) = \begin{cases} \frac{1}{9}x^2, & x \in [0,3], \\ 0, & \text{otherwise.} \end{cases}$.

a) Plot $f_X(x)$

$f_X(3) = \frac{1}{9} \times 3^2 = 1$



b) Find $P[1 < X < 2]$

$$P[1<X<2] = \int_1^2 f_X(x)\,dx = \int_1^2 \frac{1}{9}x^2\,dx = \frac{1}{9}\frac{x^3}{3}\Big|_1^2 = \frac{8-1}{27} = \frac{7}{27}$$

c) Find $P[X < 1]$

$$P[X<1] = P[-\infty < X < 1] = \int_{-\infty}^{1} f_X(x)\,dx = \int_{-\infty}^{0} \overbrace{f_X(x)}^{0}\,dx + \int_0^1 f_X(x)\,dx$$

$$= 0 + \int_0^1 \frac{1}{9}x^2\,dx = \frac{1}{27}x^3\Big|_0^1 = \frac{1}{27}$$

d) Find $P[X > 4]$

$$P[X>4] = P[4<X<\infty] = \int_4^{\infty} f_X(x)\,dx = \int_4^{\infty} 0\,dx = 0$$