

## HW 9 — Due: November 8, 5 PM

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**Instructions**

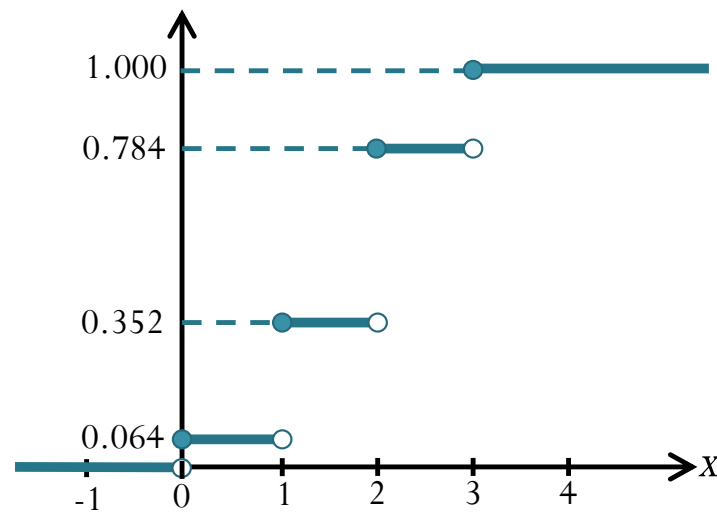
- (a) This assignment has 4 pages.
- (b) (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of *every* submitted sheet.
- (c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.
- (d) (8 pt) It is important that you try to solve all problems.
- (e) Late submission will be heavily penalized.

**Problem 1.** [F2013/1] For each of the following random variables, find  $P[1 < X \leq 2]$ .

(a)  $X \sim \text{Binomial}(3, 1/3)$

(b)  $X \sim \text{Poisson}(3)$

**Problem 2.** Arrivals of customers at the local supermarket are modeled by a Poisson process with a rate of  $\lambda = 2$  customers per minute. Let  $M$  be the number of customers arriving between 9:00 and 9:05. What is the probability that  $M < 2$ ?

Figure 9.1: CDF of  $X$  for Problem 3

**Problem 3.** [M2011/1] The cdf of a random variable  $X$  is plotted in Figure 9.1.

(a) Find the pmf  $p_X(x)$ .

(b) Find the family to which  $X$  belongs. (Uniform, Bernoulli, Binomial, Geometric, Poisson, etc.)

**Problem 4.** When  $n$  is large, binomial distribution  $\text{Binomial}(n, p)$  becomes difficult to compute directly. In this question, we will consider an approximation when the value of  $p$

is close to 0. In such case, the binomial can be approximated<sup>1</sup> by the Poisson distribution with parameter  $\alpha = np$ . For this approximation to work, we will see in this exercise that  $n$  does not have to be very large and  $p$  does not need to be very small.

- (a) Let  $X \sim \text{Binomial}(12, 1/36)$ . (For example, roll two dice 12 times and let  $X$  be the number of times a double 6 appears.) Evaluate  $p_X(x)$  for  $x = 0, 1, 2$ .

- (b) Compare your answers part (a) with its Poisson approximation.

**Problem 5.** You go to a party with 500 guests. What is the probability that exactly one other guest has the same birthday as you? Calculate this exactly and also approximately by using the Poisson pmf. (For simplicity, exclude birthdays on February 29.) [Bertsekas and Tsitsiklis, 2008, Q2.2.2]

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<sup>1</sup>More specifically, suppose  $X_n$  has a binomial distribution with parameters  $n$  and  $p_n$ . If  $p_n \rightarrow 0$  and  $np_n \rightarrow \alpha$  as  $n \rightarrow \infty$ , then

$$P[X_n = k] \rightarrow e^{-\alpha} \frac{\alpha^k}{k!}.$$

Don't forget to write your first name and the last three digit of your student ID on the upper-right corner of each submitted sheet.

## Extra Questions

Here are some optional questions for those who want more practice.

**Problem 6.** A sample of a radioactive material emits particles at a rate of 0.7 per second. Assuming that these are emitted in accordance with a Poisson distribution, find the probability that in one second

- (a) exactly one is emitted,
- (b) more than three are emitted,
- (c) between one and four (inclusive) are emitted

[Applebaum, 2008, Q5.27].

**Problem 7** (M2011/1). You are given an unfair coin with probability of obtaining a heads equal to  $1/3,000,000,000$ . You toss this coin  $6,000,000,000$  times. Let  $A$  be the event that you get “tails for all the tosses”. Let  $B$  be the event that you get “heads for all the tosses”.

- (a) Approximate  $P(A)$ .
- (b) Approximate  $P(A \cup B)$ .