ECS 315: Probability and Random Processes	**2016/1**

$$\text{HW 4 — Due: Sep 20, 5 PM}$$

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Instructions

(a) This assignment has 4 pages.

(b) (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of **every** submitted page.

(c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.

(d) (8 pt) It is important that you try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

Problem 1. Let A and B be events for which $P(A)$, $P(B)$, and $P(A \cup B)$ are known. Express the following probabilities in terms of the three known probabilities above.

(a) $P(A \cap B)$

(b) $P(A \cap B^c)$

(c) $P\left(B \cup (A \cap B^c)\right)$

(d) $P(A^c \cap B^c)$

Problem 2. Continue from Problem 3 in HW3.

Recall that, there, we consider a random experiment whose sample space is $\{a, b, c, d, e\}$ with probabilities 0.1, 0.1, 0.2, 0.4, and 0.2, respectively. Let A denote the event $\{a, b, c\}$, and let B denote the event $\{c, d, e\}$. Find the following probabilities.

(a) $P(A|B)$

(b) $P(B|A)$

(c) $P(B|A^c)$

Problem 3.

(a) Suppose that $P(A|B) = 0.4$ and $P(B) = 0.5$ Determine the following:

    (i) $P(A \cap B)$

    (ii) $P(A^c \cap B)$

[Montgomery and Runger, 2010, Q2-105]

(b) Suppose that $P(A|B) = 0.2$, $P(A|B^c) = 0.3$ and $P(B) = 0.8$ What is $P(A)$? [Montgomery and Runger, 2010, Q2-106]

Problem 4. Someone has rolled a fair dice twice. Suppose he tells you that "one of the rolls turned up a face value of six". What is the probability that the other roll turned up a six as well? [Tijms, 2007, Example 8.1, p. 244]

Hint: Note the followings:

- The answer is not $\frac{1}{6}$.

- Although there is no use of the word "give" or "conditioned on" in this question, the probability we seek is a conditional one. We have an extra piece of information because we know that the event "one of the rolls turned up a face value of six" has occurred.

- The question says "one of the rolls" without telling us which roll (the first or the second) it is referring to.

Don't forget to write your first name and the last three digit of your student ID on the upper-right corner of **every** submitted page.

Extra Question

Here is an optional question for those who want more practice.

Problem 5.

(a) Suppose that $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$. Find the range of possible values for $P(A \cap B)$. Hint: Smaller than the interval $[0, 1]$. [Capinski and Zastawniak, 2003, Q4.21]

(b) Suppose that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. Find the range of possible values for $P(A \cup B)$. Hint: Smaller than the interval $[0, 1]$. [Capinski and Zastawniak, 2003, Q4.22]