ECS 315: Probability and Random Processes **2016/1**

HW 12 — Due: November 29, 5 PM

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Instructions

(a) This assignment has 4 pages.

(b) (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of ***every*** submitted sheet.

(c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.

(d) (8 pt) It is important that you try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

Problem 1 (Yates and Goodman, 2005, Q3.3.4)**.** The pdf of random variable Y is

$$f_Y(y) = \begin{cases} y/2 & 0 \le y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $\mathbb{E}[Y]$.

(b) Find $\operatorname{Var} Y$.

Problem 2 (Yates and Goodman, 2005, Q3.3.6)**.** The cdf of random variable V is

$$F_V(v) = \begin{cases} 0 & v < -5, \\ (v+5)^2/144, & -5 \le v < 7, \\ 1 & v \ge 7. \end{cases}$$

(a) What is $f_V(v)$?

(b) What is $\mathbb{E}[V]$?

(c) What is $\mathrm{Var}[V]$?

(d) What is $\mathbb{E}[V^3]$?

Problem 3 (Yates and Goodman, 2005, Q3.4.5)**.** X is a continuous uniform RV on the interval $(-5, 5)$.

(a) What is its pdf $f_X(x)$?

(b) What is its cdf $F_X(x)$?

(c) What is $\mathbb{E}[X]$?

(d) What is $\mathbb{E}[X^5]$?

(e) What is $\mathbb{E}\left[e^X\right]$?

Problem 4 (Randomly Phased Sinusoid)**.** Suppose Θ is a uniform random variable on the interval $(0, 2\pi)$.

(a) Consider another random variable X defined by

$$X = 5\cos(7t + \Theta)$$

where t is some constant. Find $\mathbb{E}[X]$.

(b) Consider another random variable Y defined by

$$Y = 5\cos(7t_1 + \Theta) \times 5\cos(7t_2 + \Theta)$$

where t_1 and t_2 are some constants. Find $\mathbb{E}[Y]$.

Don't forget to write your first name and the last three digit of your student ID on the upper-right corner of each submitted sheet.

Extra Question

Here is an optional question for those who want more practice.

Problem 5. Let X be a uniform random variable on the interval $[0, 1]$. Set

$$A = \left[0, \frac{1}{2}\right), \quad B = \left[0, \frac{1}{4}\right) \cup \left[\frac{1}{2}, \frac{3}{4}\right), \quad \text{and } C = \left[0, \frac{1}{8}\right) \cup \left[\frac{1}{4}, \frac{3}{8}\right) \cup \left[\frac{1}{2}, \frac{5}{8}\right) \cup \left[\frac{3}{4}, \frac{7}{8}\right).$$

Are the events $[X \in A], [X \in B]$, and $[X \in C]$ independent?