ECS 315: In-Class Exercise Solution

Instructions

1. Separate into groups of no more than three persons.
2. **The group cannot be the same as your former group.**
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
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Name	ID
Prapun	555

Consider a random variable whose pmf is given by $p_X(x) = \begin{cases} \frac{c}{x^2}, & x = -2, 1, 3, \\ 0, & \text{otherwise.} \end{cases}$

$\frac{9+36+4}{36} = \frac{49}{36}$

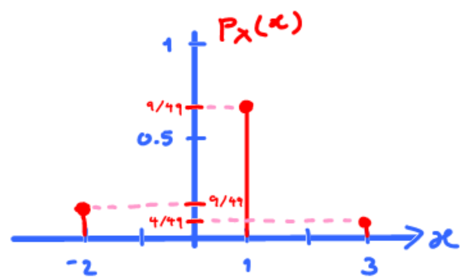
a) Find the constant c.

We know that $\sum_x p_X(x) = 1$. So, $\frac{c}{(-2)^2} + \frac{c}{1^2} + \frac{c}{3^2} = 1 \Rightarrow c\left(\frac{1}{4} + 1 + \frac{1}{9}\right) = 1$

Therefore $c = \frac{36}{49}$.

b) Plot $p_X(x)$. (Recall that we use stem plot for pmf.)

x	$p_X(x) = \frac{c}{x^2}$
-2	$c/4 = 9/49 \approx 0.1837$
1	$c/1 = 36/49 \approx 0.7347$
3	$c/9 = 4/49 \approx 0.0816$



c) Find $P\left[\left|X^2 - 5\right| < 2\right]$.

x	$\lvert x^2-5\rvert$	$\lvert x^2-5\rvert < 2$
-2	$\lvert 4-5\rvert = 1$	Yes
1	$\lvert 1-5\rvert = 4$	No
3	$\lvert 9-5\rvert = 4$	No

Therefore, $P\left[\left|X^2-5\right| < 2\right] = p_X(-2)$

$= 9/49 \approx 0.1837$

Alternatively, we can try to solve $|x^2-5| < 2$ for x.

$\equiv -2 < x^2 - 5 < 2$

$\equiv 3 < x^2 < 7$ ← probably not too useful going beyond this

$x^2 < 7 \equiv -\sqrt{7} < x < \sqrt{7}$

$x^2 > 3 \equiv x > \sqrt{3}$ or $x < \sqrt{3}$

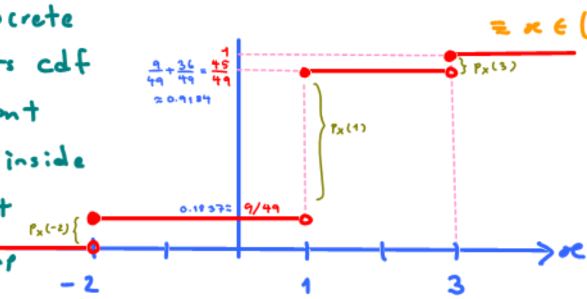
$-\sqrt{7}$ $-\sqrt{3}$ 0 $\sqrt{3}$ $\sqrt{7}$

$\equiv x \in (-\sqrt{7}, -\sqrt{3}) \cup (\sqrt{3}, \sqrt{7})$

Only $x = -2 = -\sqrt{4}$ is in the above intervals.

d) Plot $F_X(x)$.

Recall that, for discrete random variable, its cdf is piecewise constant with jumps at x inside the (minimal) support. The size of the jump at x is $p_X(x)$.



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Consider the random variable specified in each part below.

i) Write down its (minimal) support.
ii) Write down its pmf.
iii) Find P[X < 1]
iv) Find P[1 < X ≤ 2]

The RVs in this exercise are all integer-valued and non-negative.

Write the answers for the probability values in the form _._ _ _ _.
For example, write 0.5 as 0.5000, write 1/3 as 0.3333.

		Support	pmf $p_X(x) =$	P[X < 1] $= P[X=0]$	P[1 < X ≤ 2] $= P[X=2]$
(a)	$X \sim \text{Uniform}(\{1,2,3,4,5\})$	$\{1,2,3,4,5\}$	$\begin{cases} 1/5, & x \in \{1,2,3,4,5\}, \\ 0, & \text{otherwise.} \end{cases}$	$=0$ 0.0000	$= 1/5$ 0.2000
(b)	$X \sim \text{Bernoulli}\left(\frac{1}{5}\right)$	$\{0,1\}$	$\begin{cases} 1/5, & x = 1, \\ 4/5, & x = 0, \\ 0, & \text{otherwise.} \end{cases}$	$=4/5$ 0.8000	$=0$ 0.0000
(c)	$X \sim \text{Binomial}\left(5,\frac{1}{5}\right)$	$\{0,1,2,3,4,5\}$	$\begin{cases} \binom{5}{x}\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{5-x}, & x \in \{0,1,2,3,4,5\}, \\ 0, & \text{otherwise.} \end{cases}$	$=(4/5)^5$ 0.3277	0.2048

$$\binom{5}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^{5-2} = 10 \times \frac{4^3}{5^5}$$

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Consider a random variable whose pmf is given by $p_X(x) = \begin{cases} \frac{c}{x^2}, & x = -2, 1, 3, \\ 0, & \text{otherwise.} \end{cases}$.

a) Check that $c = \frac{36}{49}$.

$$\sum_x p_X(x) = p_X(-2) + p_X(1) + p_X(3) = \frac{36}{49}\times\frac{1}{4} + \frac{36}{49}\times 1 + \frac{36}{49}\times\frac{1}{9} = \frac{36}{49}\left(\frac{9}{36}+\frac{36}{36}+\frac{4}{36}\right)$$

$$= \frac{36}{49}\times\frac{49}{36} = 1 \checkmark$$

b) Find $\mathbb{E}X$

$$\mathbb{E}X = \sum_x x\, p_X(x) = \left((-2)\times\frac{1}{4} + (1)\times 1 + (3)\times\frac{1}{9}\right)\times\frac{36}{49} = \left(-\frac{1}{2}+1+\frac{1}{3}\right)\times\frac{36}{49}$$

$$= \frac{5}{\cancel{6}}\times\frac{\cancelto{6}{36}}{49} = \frac{30}{49} \approx 0.6122$$

c) Let $Y = (X-2)^2$. $\quad Y = g(X)$ where $g(x) = (x-2)^2$

a. Find $p_Y(y)$.

$p_X(x)$	x	$y = (x-2)^2$
$c/4$	-2	$4^2 = 16$
$c/1$	1	$(-1)^2 = 1$
$c/9$	3	$1^2 = 1$

$S_Y = \{1, 16\}$

$$p_Y(1) = p_X(1) + p_X(3) = \left(1+\frac{1}{9}\right)\times\frac{36}{49} = \frac{40}{49}$$

$$p_Y(16) = p_X(-2) = \frac{1}{4}\times\frac{36}{49} = \frac{9}{49}$$

$$p_Y(y) = \begin{cases} 40/49 \ (\approx 0.8163), & y = 1, \\ 9/49 \ (\approx 0.0204), & y = 16, \\ 0, & \text{otherwise.} \end{cases}$$

b. Find $\mathbb{E}Y$.

$$\mathbb{E}Y = \sum_y y\, p_Y(y) = 1\times\frac{40}{49} + 16\times\frac{9}{49} = \frac{184}{49} \approx 3.7551$$

$$\mathbb{E}Y = \mathbb{E}\left[(X-2)^2\right] = \mathbb{E}\left[X^2 - 4X + 4\right] = \mathbb{E}[X^2] - 4\mathbb{E}X + 4.$$

To find $\mathbb{E}[X^2]$. Let $Z = X^2$

$p_X(x)$	x	$z = x^2$
$c/4$	-2	4
$c/1$	1	1
$c/9$	3	9

So,

$$p_Z(z) = \begin{cases} c/4, & z = 4, \\ c, & z = 1, \\ c/9, & z = 9. \end{cases}$$

$$\mathbb{E}Z = \frac{c}{4} \times 4 + c \times 1 + \frac{c}{9} \times 9 = 3c$$

same

Alternatively, from LOTUS,

$$\mathbb{E}[X^2] = \sum_x x^2 p_X(x) = \sum_x \cancel{x^2} \times \frac{c}{\cancel{x^2}} = 3c$$

Therefore, $\mathbb{E}Y = 3 \times \frac{36}{49} - 4 \times \frac{30}{49} + 4 \approx 3.7551$ (same as above)

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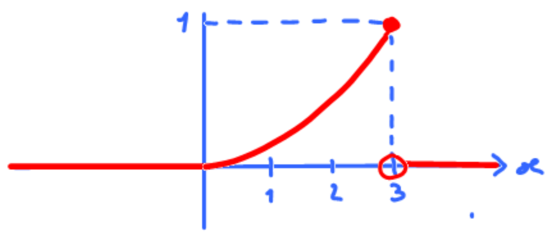
Consider a continuous random variable whose pdf is given by $f_X(x) = \begin{cases} \frac{1}{9}x^2, & x \in [0,3], \\ 0, & \text{otherwise.} \end{cases}$.

a) Plot $f_X(x)$

$f_X(3) = \frac{1}{9} \times 3^2 = 1$



b) Find $P[1 < X < 2]$

$$P[1<X<2] = \int_1^2 f_X(x)\,dx = \int_1^2 \frac{1}{9}x^2\,dx = \frac{1}{9}\frac{x^3}{3}\Big|_1^2 = \frac{8-1}{27} = \frac{7}{27}$$

c) Find $P[X < 1]$

$$P[X<1] = P[-\infty < X < 1] = \int_{-\infty}^{1} f_X(x)\,dx = \int_{-\infty}^{0} \overbrace{f_X(x)}^{0}\,dx + \int_0^1 f_X(x)\,dx$$

$$= 0 + \int_0^1 \frac{1}{9}x^2\,dx = \frac{1}{27}x^3\Big|_0^1 = \frac{1}{27}$$

d) Find $P[X > 4]$

$$P[X>4] = P[4<X<\infty] = \int_4^{\infty} f_X(x)\,dx = \int_4^{\infty} 0\,dx = 0$$