

HW 9 — Due: Nov 11, 9:19 AM (in tutorial session)

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Instructions

- (a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.
- (b) It is important that you try to solve all problems. (5 pt)
The extra questions at the end are optional.
- (c) Late submission will be rejected.

Problem 1 (Modified from Yates and Goodman, 2005, Q3.1.3). The CDF of a random variable W is

$$F_W(w) = \begin{cases} 0, & w < -5, \\ (w + 5)/8, & -5 \leq w < -3, \\ 1/4, & -3 \leq w < 3, \\ 1/4 + 3(w - 3)/8, & 3 \leq w < 5, \\ 1, & w \geq 5. \end{cases}$$

- (a) Is W a continuous random variable?
- (b) What is $P[W \leq 4]$?
- (c) What is $P[-2 < W \leq 2]$?
- (d) What is $P[W > 0]$?
- (e) What is the value of a such that $P[W \leq a] = 1/2$?

Problem 2 (Yates and Goodman, 2005, Q3.2.1). The random variable X has probability density function

$$f_X(x) = \begin{cases} cx & 0 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Use the pdf to find

- (a) the constant c ,

- (b) $P[0 \leq X \leq 1]$,
- (c) $P[-1/2 \leq X \leq 1/2]$,
- (d) the cdf $F_X(x)$.

Problem 3 (Yates and Goodman, 2005, Q3.2.3). The CDF of random variable W is

$$F_W(w) = \begin{cases} 0, & w < -5, \\ (w + 5)/8, & -5 \leq w < -3, \\ 1/4, & -3 \leq w < 3, \\ 1/4 + 3(w - 3)/8, & 3 \leq w < 5, \\ 1, & w \geq 5. \end{cases}$$

Find its pdf $f_W(w)$.

Problem 4 (Yates and Goodman, 2005, Q3.3.4). The pdf of random variable Y is

$$f_Y(y) = \begin{cases} y/2 & 0 \leq y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

What are $\mathbb{E}[Y]$ and $\text{Var} Y$?

Problem 5 (Yates and Goodman, 2005, Q3.3.6). The cdf of random variable V is

$$F_V(v) = \begin{cases} 0 & v < -5, \\ (v + 5)^2/144, & -5 \leq v < 7, \\ 1 & v \geq 7. \end{cases}$$

- (a) What is $\mathbb{E}[V]$?
- (b) What is $\text{Var}[V]$?
- (c) What is $\mathbb{E}[V^3]$?