ECS 315: Probability and Random Processes **2015/1**

HW 6 — Due: Not Due

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Problem 1. An optical inspection system is to distinguish among different part types. The probability of a correct classification of any part is 0.98. Suppose that three parts are inspected and that the classifications are independent.

(a) Let the random variable X denote the number of parts that are correctly classified. Determine the probability mass function of X. [Montgomery and Runger, 2010, Q3-20]

(b) Let the random variable Y denote the number of parts that are incorrectly classified. Determine the probability mass function of Y.

Problem 2. Consider the sample space $\Omega = \{-2, -1, 0, 1, 2, 3, 4\}$. Suppose that $P(A) = |A|/|\Omega|$ for any event $A \subset \Omega$. Define the random variable $X(\omega) = \omega^2$. Find the probability mass function of X.

Problem 3. Suppose X is a random variable whose pmf at $x = 0, 1, 2, 3, 4$ is given by $p_X(x) = \frac{2x+1}{25}$.

Remark: Note that the statement above does not specify the value of the $p_X(x)$ at the value of x that is not 0,1,2,3, or 4.

(a) What is $p_X(5)$?

(b) Determine the following probabilities:

(i) $P[X = 4]$

(ii) $P[X \leq 1]$

(iii) $P[2 \leq X < 4]$

(iv) $P[X > -10]$

Problem 4. The random variable V has pmf

$$p_V(v) = \begin{cases} cv^2, & v = 1, 2, 3, 4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant c.

(b) Find $P[V \in \{u^2 : u = 1, 2, 3, \ldots\}]$.

(c) Find the probability that V is an even number.

(d) Find $P[V > 2]$.

(e) Sketch $p_V(v)$.

(f) Sketch $F_V(v)$. (Note that $F_V(v) = P[V \le v]$.)

Problem 5. The thickness of the wood paneling (in inches) that a customer orders is a random variable with the following cdf:

$$F_X(x) = \begin{cases} 0, & x < \frac{1}{8} \\ 0.2, & \frac{1}{8} \le x < \frac{1}{4} \\ 0.9, & \frac{1}{4} \le x < \frac{3}{8} \\ 1 & x \ge \frac{3}{8} \end{cases}$$

Determine the following probabilities:

(a) $P[X \le 1/18]$

(b) $P[X \le 1/4]$

(c) $P[X \le 5/16]$

(d) $P[X > 1/4]$

(e) $P[X \le 1/2]$

[Montgomery and Runger, 2010, Q3-42]

Extra Questions

Here are some optional questions for those who want more practice.

Problem 6. Consider a transmission over a binary symmetric channel (BSC) with crossover probability p. The random (binary) input to the BSC is denoted by X. Let p_1 be the probability that $X = 1$. (This implies the probability that $X = 0$ is $1 - p_1$.) Let Y by the output of the BSC.

(a) Suppose, at the receiver (which observes the output of the BSC), we learned that $Y = 1$. For each of the following scenarios, which event is more likely, "$X = 1$ was transmitted" or "$X = 0$ was transmitted"? (Hint: Use Bayes' theorem.)

(i) Assume $p = 0.3$ and $p_1 = 0.1$.

(ii) Assume $p = 0.3$ and $p_1 = 0.5$.

(iii) Assume $p = 0.3$ and $p_1 = 0.9$.

(iv) Assume $p = 0.7$ and $p_1 = 0.5$.

(b) Suppose, at the receiver (which observes the output of the BSC), we learned that $Y = 0$. For each of the following scenarios, which event is more likely, "$X = 1$ was transmitted" or "$X = 0$ was transmitted"?

(i) Assume $p = 0.3$ and $p_1 = 0.1$

(ii) Assume $p = 0.3$ and $p_1 = 0.5$

(iii) Assume $p = 0.3$ and $p_1 = 0.9$

(iv) Assume $p = 0.7$ and $p_1 = 0.5$

Remark: A MAP (maximum a posteriori) detector is a detector that takes the observed value Y and then calculate the most likely transmitted value. More specifically,

$$\hat{x}_{MAP}(y) = \arg\max_{x} P\left[X = x | Y = y\right]$$

In fact, in part (a), each of your answers is $\hat{x}_{MAP}(1)$ and in part (b), each of your answers is $\hat{x}_{MAP}(0)$.