

HW 13 — Due: Not Due

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Problem 1. The input X and output Y of a system subject to random perturbations are described probabilistically by the following joint pmf matrix:

$x \backslash y$	2	4	5
1	[0.02	0.10
3	[0.08	0.32
]]]

(a) Evaluate the following quantities:

- (i) $\mathbb{E}[XY]$
- (ii) $\mathbb{E}[(X - 3)(Y - 2)]$
- (iii) $\mathbb{E}[X(Y^3 - 11Y^2 + 38Y)]$
- (iv) $\text{Cov}[X, Y]$
- (v) $\rho_{X,Y}$

Hint: Write down the formulas then use MATLAB or Excel to compute them.

(b) Find $\rho_{X,X}$

(c) Calculate the following quantities using the values of $\text{Var } X$, $\text{Cov}[X, Y]$, and $\rho_{X,Y}$ that you got earlier.

- (i) $\text{Cov}[3X + 4, 6Y - 7]$
- (ii) $\rho_{3X+4, 6Y-7}$
- (iii) $\text{Cov}[X, 6X - 7]$
- (iv) $\rho_{X, 6X-7}$

Problem 2. Suppose $X \sim \text{binomial}(5, 1/3)$, $Y \sim \text{binomial}(7, 4/5)$, and $X \perp\!\!\!\perp Y$. Evaluate the following quantities.

- (a) $\mathbb{E}[(X - 3)(Y - 2)]$
- (b) $\text{Cov}[X, Y]$
- (c) $\rho_{X,Y}$

Problem 3. Suppose we know that $\sigma_X = \frac{\sqrt{21}}{10}$, $\sigma_Y = \frac{4\sqrt{6}}{5}$, $\rho_{X,Y} = -\frac{1}{\sqrt{126}}$.

- (a) Find $\text{Var}[X + Y]$.
- (b) Find $\mathbb{E}[(Y - 3X + 5)^2]$. Assume $\mathbb{E}[Y - 3X + 5] = 1$.

Problem 4. The input X and output Y of a system subject to random perturbations are described probabilistically by the joint pmf $p_{X,Y}(x, y)$, where $x = 1, 2, 3$ and $y = 1, 2, 3, 4, 5$. Let P denote the joint pmf matrix whose i, j entry is $p_{X,Y}(i, j)$, and suppose that

$$P = \frac{1}{71} \begin{bmatrix} 7 & 2 & 8 & 5 & 4 \\ 4 & 2 & 5 & 5 & 9 \\ 2 & 4 & 8 & 5 & 1 \end{bmatrix}$$

- (a) Find the marginal pmfs $p_X(x)$ and $p_Y(y)$.
- (b) Find $\mathbb{E}X$
- (c) Find $\mathbb{E}Y$
- (d) Find $\text{Var} X$
- (e) Find $\text{Var} Y$

Problem 5. A webpage server can handle r requests per day. Find the probability that the server gets more than r requests at least once in n days. Assume that the number of requests on day i is $X_i \sim \mathcal{P}(\alpha)$ and that X_1, \dots, X_n are independent.

Extra Questions

Here are some optional questions for those who want more practice.

Problem 6. Suppose $X \sim \text{binomial}(5, 1/3)$, $Y \sim \text{binomial}(7, 4/5)$, and $X \perp\!\!\!\perp Y$.

- (a) A vector describing the pmf of X can be created by the MATLAB expression:

$$\mathbf{x} = 0:5; \mathbf{pX} = \text{binopdf}(\mathbf{x}, 5, 1/3).$$

What is the expression that would give \mathbf{pY} , a corresponding vector describing the pmf of Y ?

- (b) Use \mathbf{pX} and \mathbf{pY} from part (a), how can you create the joint pmf matrix in MATLAB? Do not use “for-loop”, “while-loop”, “if statement”. Hint: Multiply them in an appropriate orientation.
- (c) Use MATLAB to evaluate the following quantities. Again, do not use “for-loop”, “while-loop”, “if statement”.
- (i) $\mathbb{E}X$
 - (ii) $P[X = Y]$
 - (iii) $P[XY < 6]$

Problem 7. Suppose $\text{Var } X = 5$. Find $\text{Cov}[X, X]$ and $\rho_{X,X}$.