ECS 315: Probability and Random Processes **2015/1**

HW 11 — Due: Nov 25, 9:19 AM

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)
The extra questions at the end are optional.

(c) Late submission will be rejected.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Suppose that the time to failure (in hours) of fans in a personal computer can be modeled by an exponential distribution with $\lambda = 0.0003$.

(a) What proportion of the fans will last at least 10,000 hours?

(b) What proportion of the fans will last at most 7000 hours?

[Montgomery and Runger, 2010, Q4-97]

Problem 2. Consider each random variable X defined below. Let $Y = 1 + 2X$. (i) Find and sketch the pdf of Y and (ii) Does Y belong to any of the (popular) families discussed in class? If so, state the name of the family and find the corresponding parameters.

(a) $X \sim \mathcal{U}(0, 1)$

(b) $X \sim \mathcal{E}(1)$

(c) $X \sim \mathcal{N}(0, 1)$

Problem 3. Consider each random variable X defined below. Let $Y = 1 - 2X$. (i) Find and sketch the pdf of Y and (ii) Does Y belong to any of the (popular) families discussed in class? If so, state the name of the family and find the corresponding parameters.

(a) $X \sim \mathcal{U}(0,1)$

(b) $X \sim \mathcal{E}(1)$

(c) $X \sim \mathcal{N}(0,1)$

Problem 4. Let $X \sim \mathcal{E}(5)$ and $Y = 2/X$.

(a) Check that Y is still a continuous random variable.

(b) Find $F_Y(y)$.

(c) Find $f_Y(y)$.

(d) (optional) Find $\mathbb{E}Y$.

Hint: Because $\frac{d}{dy}e^{-\frac{10}{y}} = \frac{10}{y^2}e^{-\frac{10}{y}} > 0$ for $y \neq 0$. We know that $e^{-\frac{10}{y}}$ is an increasing function on our range of integration. In particular, consider $y > 10/\ln(2)$. Then, $e^{-\frac{10}{y}} > \frac{1}{2}$. Hence,

$$\int_0^\infty \frac{10}{y}e^{-\frac{10}{y}}dy > \int_{10/\ln 2}^\infty \frac{10}{y}e^{-\frac{10}{y}}dy > \int_{10/\ln 2}^\infty \frac{10}{y}\frac{1}{2}dy = \int_{10/\ln 2}^\infty \frac{5}{y}dy.$$

Remark: To be technically correct, we should be a little more careful when writing $Y = \frac{2}{X}$ because it is undefined when $X = 0$. Of course, this happens with 0 probability; so it won't create any serious problem. However, to avoid the problem, we may define Y by

$$Y = \begin{cases} 2/X, & X \neq 0, \\ 0, & X = 0. \end{cases}$$

Problem 5. In wireless communications systems, fading is sometimes modeled by ***lognormal*** random variables. We say that a positive random variable Y is lognormal if $\ln Y$ is a normal random variable (say, with expected value m and variance σ^2).

(a) Check that Y is still a continuous random variable.

(b) Find the pdf of Y.

Hint: First, recall that the ln is the natural log function (log base e). Let $X = \ln Y$. Then, because Y is lognormal, we know that $X \sim \mathcal{N}(m, \sigma^2)$. Next, write Y as a function of X.

Extra Questions

Here are some optional questions for those who want more practice.

Problem 6. Cholesterol is a fatty substance that is an important part of the outer lining (membrane) of cells in the body of animals. Its normal range for an adult is 120–240 mg/dl. The Food and Nutrition Institute of the Philippines found that the total cholesterol level for Filipino adults has a mean of 159.2 mg/dl and 84.1% of adults have a cholesterol level below 200 mg/dl. Suppose that the cholesterol level in the population is normally distributed.

(a) Determine the standard deviation of this distribution.

(b) What is the value of the cholesterol level that exceeds 90% of the population?

(c) An adult is at moderate risk if cholesterol level is more than one but less than two standard deviations above the mean. What percentage of the population is at moderate risk according to this criterion?

(d) An adult is thought to be at high risk if his cholesterol level is more than two standard deviations above the mean. What percentage of the population is at high risk?

Problem 7. Consider a random variable X whose pdf is given by

$$f_X(x) = \begin{cases} cx^2, & x \in (1,2), \\ 0, & \text{otherwise.} \end{cases}$$

Let $Y = 4|X - 1.5|$.

(a) Find $\mathbb{E}Y$.

(b) Find $f_Y(y)$.