Example 6.10. Consider the following sequences of 1s and 0s which summarize the data obtained from 15 testees.

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D:  0 1 1 0 0 0 0 1 1 1 1 0 1 0 1

TP: 1 0 0 1 1 0 0 0 0 0 1 1 0 1 1
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The "D" row indicates whether each of the testees actually has the disease under investigation. The "TP" row indicates whether each of the testees is tested positive for the disease.

Numbers "1" and "0" correspond to "True" and "False", respectively.

Suppose we randomly pick a testee from this pool of 15 persons. Let D be the event that this selected person actually has the disease. Let T_P be the event that this selected person is tested positive for the disease.

Find the following probabilities.

(a) $P(D) = \frac{8}{15}$ Among the 15 testees, 8 have the disease.

(b) $P(D^c) = \frac{7}{15}$ Among the 15 testees, 7 do not have the disease.

(c) $P(T_P) = \frac{7}{15}$ Among the 15 testees, 7 test positive.

(d) $P(T_P^c) = \frac{8}{15}$ Among the 15 testees, 8 test negative.

(e) $P(T_P|D) = \frac{2}{8} = \frac{1}{4}$ Among the 8 testees who have the disease, two test positive.

(f) $P(T_P|D^c) = \frac{5}{7}$ Among the 7 testees who don't have the disease, 5 test positive.

(g) $P(T_P^c|D) = \frac{6}{8} = \frac{3}{4}$ Among the 8 testees who have the disease, 6 test negative.

(h) $P(T_P^c|D^c) = \frac{2}{7}$ Among the 7 testees who don't have the disease, 2 test positive.

ECS 315: Quiz 2 Solution

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. *Write down all the steps* that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. Do not panic.

Name	ID
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Consider events A, B, C, D defined on a sample space Ω.

Suppose

$$P(B) = 1/3,\ P(C) = 2/3,\ P(D) = 1/4,$$

$$P(A|B) = 1/5,\ P(A|C) = 3/5,\ P(A|D) = 1.$$

(a) Find $P(A \cap B)$.

$$P(A\cap B) = P(A|B)P(B) = \frac{1}{5}\times\frac{1}{3} = \frac{1}{15}$$

(b) The collection {B, C, D} can *not* be a partition of Ω. Why?

To be a partition, they must be disjoint and their union must $= \Omega$.

This implies $P(B\cup C\cup D)$ must $= P(\Omega) = 1$.

However, when they are disjoint, $P(B\cup C\cup D) = P(B)+P(C)+P(D)$

$$= \frac{1}{3}+\frac{2}{3}+\frac{1}{4} = \frac{5}{4} \neq 1$$

contradiction

(c) Suppose $B = C^c$.

a. Use the total probability theorem to find P(*A*).

First, note that because $B = C^c$, the collection $\{B, C\}$ forms a partion of the sample space.

Therefore, $P(A) = P(A|B)P(B) + P(A|C)P(C) = \frac{1}{5}\times\frac{1}{3} + \frac{3}{5}\times\frac{2}{3} = \frac{1}{15}+\frac{6}{15} = \frac{7}{15}$

b. Find P(B|A).

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{\frac{1}{5}\times\frac{1}{3}}{\frac{7}{15}} = \frac{1}{7}$$

Alternatively, because we already know that $P(A\cap B) = \frac{1}{15}$ from the first part, we may find $P(B|A)$ directly from its definition:

$$P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{1/15}{7/15} = \frac{1}{7} \quad \text{(same as above)}$$

ECS 315: Quiz 3 Sol

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. *Write down all the steps* that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. Do not panic.

Name	ID
Prapun	555

1. Suppose $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{3}$. Find $P(A\cap B)$ to make A and B independent.

To make $A \perp\!\!\!\perp B$, we need $P(A\cap B) = P(A)P(B)$

Therefore, $P(A\cap B) = \frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$.

2. Suppose $P(C)=\frac{1}{3}$ and $P(D\cap C^c)=\frac{1}{6}$. Find $P(C\cap D)$ to make C and D independent.

3. Consider a digital transmission through a binary symmetric channel with crossover probability p = 0.3. No coding is used.
   Suppose we input a sequence of three bits 101 into the channel.
   a. Find the probability that we will get 100 as its output.

only the last bit is in error

last bit in error

$$\text{probability} = \underbrace{(1-p)}_{\text{first bit not in error}} \times \underbrace{(1-p)}_{\text{second bit not in error}} \times p = (0.7)^2 \times 0.3 = 0.147$$

   b. Find the probability that exactly one of the three bits is in error at the channel output.

k n

$$\text{probability} = \binom{n}{k} p^k (1-p)^{n-k} = \binom{3}{1} 0.3^1 \times 0.7^2 = 3\times 0.147$$

$$= 0.441$$