

ECS 315: Quiz 4

Consider the random variable specified in each part below.

- i) Write down its (minimal) support.
- ii) Write down its pmf.
- iii) Find $P[X < 1]$
- iv) Find $P[1 < X \leq 2]$

		Support	pmf	$P[X < 1]$	$P[1 < X \leq 2]$
(a)	$X \sim \text{Bernoulli}\left(\frac{1}{2}\right)$	$\{0, 1\}$	$\begin{cases} 1/2, & x=0, 1, \\ 0, & \text{otherwise} \end{cases}$	$1/2$	0
(b)	$X \sim \text{Binomial}\left(4, \frac{1}{4}\right)$	$\{0, 1, 2, 3, 4\}$	$\begin{cases} \binom{4}{x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{4-x}, & x=0, 1, 2, 3, 4 \\ 0, & \text{otherwise} \end{cases}$	$\frac{81}{256} \approx 0.3164$	$\frac{27}{128} \approx 0.2109$
(c)	$X \sim \text{Poisson}(1)$	$\{0, 1, 2, \dots\}$	$\begin{cases} \frac{1}{e x!}, & x=0, 1, 2, \dots \\ 0, & \text{otherwise} \end{cases}$	$\frac{1}{e} \approx 0.3679$	$\frac{1}{2e} \approx 0.1839$

$X \sim \text{Bernoulli}(p) \Rightarrow \text{pmf } p_x(x) = \begin{cases} 1-p, & x=0, \\ p, & x=1, \\ 0, & \text{otherwise.} \end{cases}$

$\downarrow p=1/2$
 $= \begin{cases} 1/2, & x=0, 1, \\ 0, & \text{otherwise} \end{cases}$

minimal support $S_x = \{0, 1\}$

$P[X < 1] = P[X=0] = 1/2$

$P[1 < X \leq 2] = P[X=2] = 0$

$X \sim \text{Binomial}(n, p) \Rightarrow \text{pmf } p_x(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x=0, 1, 2, \dots, n, \\ 0, & \text{otherwise.} \end{cases}$

$n=4, p=1/4 \Rightarrow \begin{cases} \binom{4}{x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{4-x}, & x=0, 1, 2, 3, 4 \\ 0, & \text{otherwise} \end{cases}$

$P[X < 1] = P[X=0] = \frac{81}{256} \approx 0.3164$
 $P[1 < X \leq 2] = P[X=2] = \frac{27}{128} \approx 0.2109$

$\begin{cases} 81/256 \approx 0.3164, & x=0, \\ 27/64 \approx 0.4219, & x=1, \\ 27/128 \approx 0.2109, & x=2, \\ 3/64 \approx 0.0468, & x=3, \\ 1/256 \approx 0.0039, & x=4 \\ 0, & \text{otherwise} \end{cases}$

minimal support $S_x = \{0, 1, 2, 3, 4\}$

$X \sim \text{Poisson}(\alpha) \Rightarrow \text{pmf } p_x(x) = \begin{cases} \frac{e^{-\alpha} \alpha^x}{x!}, & x=0, 1, 2, \dots \\ 0, & \text{otherwise} \end{cases}$

$\uparrow \alpha=1$
 $= \begin{cases} \frac{1}{e x!}, & x=0, 1, 2, \dots \\ 0, & \text{otherwise.} \end{cases}$

min. support $S_x = \{0, 1, 2, \dots\}$

$P[X < 1] = P[X=0] = \frac{1}{e 0!} = \frac{1}{e} \approx 0.3679$

$P[1 < X \leq 2] = P[X=2] = \frac{1}{e \times 2!} = \frac{1}{2e} \approx 0.1839$