ECS 315: Quiz 3 Sol

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Name	ID
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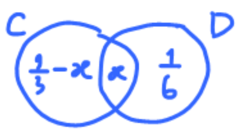
1. Suppose $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{3}$. Find $P(A\cap B)$ to make A and B independent.

To make $A \perp\!\!\!\perp B$, we need $P(A\cap B) = P(A)P(B)$

Therefore, $P(A\cap B) = \frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$.

2. Suppose $P(C)=\frac{1}{3}$ and $P(D\cap C^c)=\frac{1}{6}$. Find $P(C\cap D)$ to make C and D independent.

To make $C \perp\!\!\!\perp D$, we need $P(C\cap D) = P(C)P(D)$. Let $P(C\cap D) = x$.



$$x = \frac{1}{3}\left(x+\frac{1}{6}\right)$$
$$3x = x+\frac{1}{6}$$
$$2x = \frac{1}{6}$$
$$x = \frac{1}{12}$$

Alternatively, we need $P(D\cap C^c) \overset{\text{Another equivalent property}}{=} P(D)P(C^c)$

$\frac{1}{6} = P(D)\left(1-\frac{1}{3}\right) \Rightarrow P(D) = \frac{1}{6}\times\frac{3}{2} = \frac{1}{4} \Rightarrow P(C\cap D) = P(D) - P(D\cap C^c) = \frac{1}{4}-\frac{1}{6} = \frac{2}{24} = \frac{1}{12}$

3. Consider a digital transmission through a binary symmetric channel with crossover probability $p = 0.3$. No coding is used.
Suppose we input a sequence of three bits 101 into the channel.
   a. Find the probability that we will get 100 as its output.

only the last bit is in error

$$\text{probability} = \underbrace{(1-p)}_{\text{first bit not in error}} \times \underbrace{(1-p)}_{\text{second bit not in error}} \times \underbrace{p}_{\text{last bit in error}} = (0.7)^2\times 0.3 = 0.147$$

   b. Find the probability that exactly one (k) of the three (n) bits is in error at the channel output.

$$\text{probability} = \binom{n}{k}p^k(1-p)^{n-k} = \binom{3}{1}0.3^1\times 0.7^2 = 3\times 0.147$$
$$= 0.441$$