ECS 315: Quiz 2 Solution

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Name	ID
Prapun	555

Consider events A, B, C, D defined on a sample space Ω.

Suppose

$$P(B) = 1/3,\ P(C) = 2/3,\ P(D) = 1/4,$$

$$P(A|B) = 1/5,\ P(A|C) = 3/5,\ P(A|D) = 1.$$

(a) Find $P(A \cap B)$.

$$P(A \cap B) = P(A|B)P(B) = \frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$$

(b) The collection {B, C, D} can ***not*** be a partition of Ω. Why?

To be a partition, they must be disjoint and their union must $= \Omega$.

This implies $P(B \cup C \cup D)$ must $= P(\Omega) = 1$.

However, when they are disjoint, $P(B \cup C \cup D) = P(B) + P(C) + P(D)$

$$= \frac{1}{3} + \frac{2}{3} + \frac{1}{4} = \frac{5}{4} \neq 1$$

contradiction

(c) Suppose $B = C^c$.

a. Use the total probability theorem to find P(A).

First, note that because $B = C^c$, the collection $\{B, C\}$ forms a partion of the sample space.

Therefore, $P(A) = P(A|B)P(B) + P(A|C)P(C) = \frac{1}{5} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{3} = \frac{1}{15} + \frac{6}{15} = \frac{7}{15}$

b. Find P(B|A).

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{\frac{1}{5} \times \frac{1}{3}}{\frac{7}{15}} = \frac{1}{7}$$

Alternatively, because we already know that $P(A \cap B) = \frac{1}{15}$ from the first part, we may find $P(B|A)$ directly from its definition:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/15}{7/15} = \frac{1}{7} \quad \text{(same as above)}$$