

Example 6.10. Consider the following sequences of 1s and 0s which summarize the data obtained from 15 testees.

D: 0 $\overset{1}{\textcircled{1}}$ $\overset{2}{\textcircled{1}}$ 0 0 0 0 $\overset{3}{\textcircled{1}}$ $\overset{4}{\textcircled{1}}$ $\overset{5}{\textcircled{1}}$ $\overset{6}{\textcircled{1}}$ 0 $\overset{7}{\textcircled{1}}$ 0 $\overset{8}{\textcircled{1}}$

\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow

TP: $\textcircled{1}_1$ 0 0 $\textcircled{1}_2$ $\textcircled{1}_3$ 0 0 0 0 0 $\textcircled{1}_4$ $\textcircled{1}_5$ 0 $\textcircled{1}_6$ $\textcircled{1}_7$

The “D” row indicates whether each of the testees actually has the disease under investigation. The “TP” row indicates whether each of the testees is tested positive for the disease.

Numbers “1” and “0” correspond to “True” and “False”, respectively.

Suppose we randomly pick a testee from this pool of 15 persons. Let D be the event that this selected person actually has the disease. Let T_P be the event that this selected person is tested positive for the disease.

Find the following probabilities.

- (a) $P(D) = \frac{8}{15}$ Among the 15 testees, 8 have the disease.
- (b) $P(D^c) = \frac{7}{15}$ Among the 15 testees, 7 do not have the disease.
- (c) $P(T_P) = \frac{7}{15}$ Among the 15 testees, 7 test positive.
- (d) $P(T_P^c) = \frac{8}{15}$ Among the 15 testees, 8 test negative.
- (e) $P(T_P|D) = \frac{2}{8} = \frac{1}{4}$ Among the 8 testees who have the disease, two test positive.
- (f) $P(T_P|D^c) = \frac{5}{7}$ Among the 7 testees who don't have the disease, 5 test positive.
- (g) $P(T_P^c|D) = \frac{6}{8} = \frac{3}{4}$ Among the 8 testees who have the disease, 6 test negative.
- (h) $P(T_P^c|D^c) = \frac{2}{7}$ Among the 7 testees who don't have the disease, 2 test positive.