

## ECS315 2015/1 Part VII Dr.Prapun

### 13 Three Types of Random Variables

**13.1.** Review: You may recall<sup>53</sup> the following properties for cdf of discrete random variables. These properties hold for any kind of random variables.

(a) The cdf is defined as  $F_X(x) = P[X \leq x]$ . This is valid for any type of random variables.

(b) Moreover, the cdf for any kind of random variable must satisfies three properties which we have discussed earlier:

CDF1  $F_X$  is non-decreasing

CDF2  $F_X$  is right-continuous

CDF3  $\lim_{x \rightarrow -\infty} F_X(x) = 0$  and  $\lim_{x \rightarrow \infty} F_X(x) = 1$ .

(c)  $P[X = x] = F_X(x) - F_X(x^-) =$  the jump or saltus in  $F$  at  $x$ .

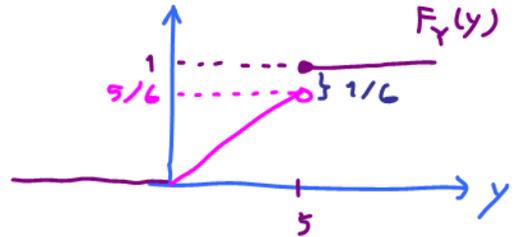
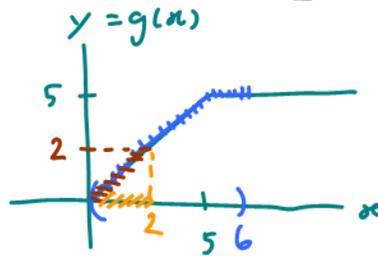
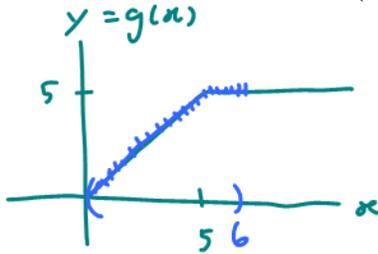
**Theorem 13.2.** If you find a function  $F$  that satisfies CDF1, CDF2, and CDF3 above, then  $F$  is a cdf of some random variable.

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<sup>53</sup>If you don't know these properties by now, you should review them as soon as possible.

$$x \rightarrow \boxed{g(\cdot)} \rightarrow Y$$

**Example 13.3.** Consider an input  $X$  to a device whose output  $Y$  will be the same as the input if the input level does not exceed 5. For input level that exceeds 5, the output will be saturated at 5. Suppose  $X \sim \mathcal{U}(0, 6)$ . Find  $F_Y(y)$ .  $P[Y=5] = \frac{1}{6}$



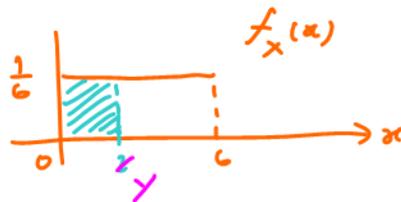
$$F_Y(y) \equiv P[Y \leq y]$$

For  $0 \leq y < 5$ , e.g.  $y=2$

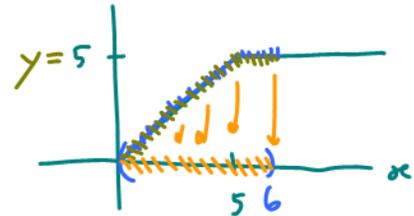
$$F_Y(2) \equiv P[Y \leq 2] = P[X \leq 2] = \frac{1}{3}$$

For  $y=5$ ,

$$F_Y(5) \equiv P[Y \leq 5] = 1$$



$$F_Y(y) \equiv P[Y \leq y] = P[X \leq y] = \frac{y}{6}$$



**13.4.** We can categorize random variables into three types according to its cdf:

- If  $F_X(x)$  is piecewise flat with discontinuous jumps, then  $X$  is **discrete**.
- If  $F_X(x)$  is a continuous function, then  $X$  is **continuous**.
- If  $F_X(x)$  is a piecewise continuous function with discontinuities, then  $X$  is **mixed**.

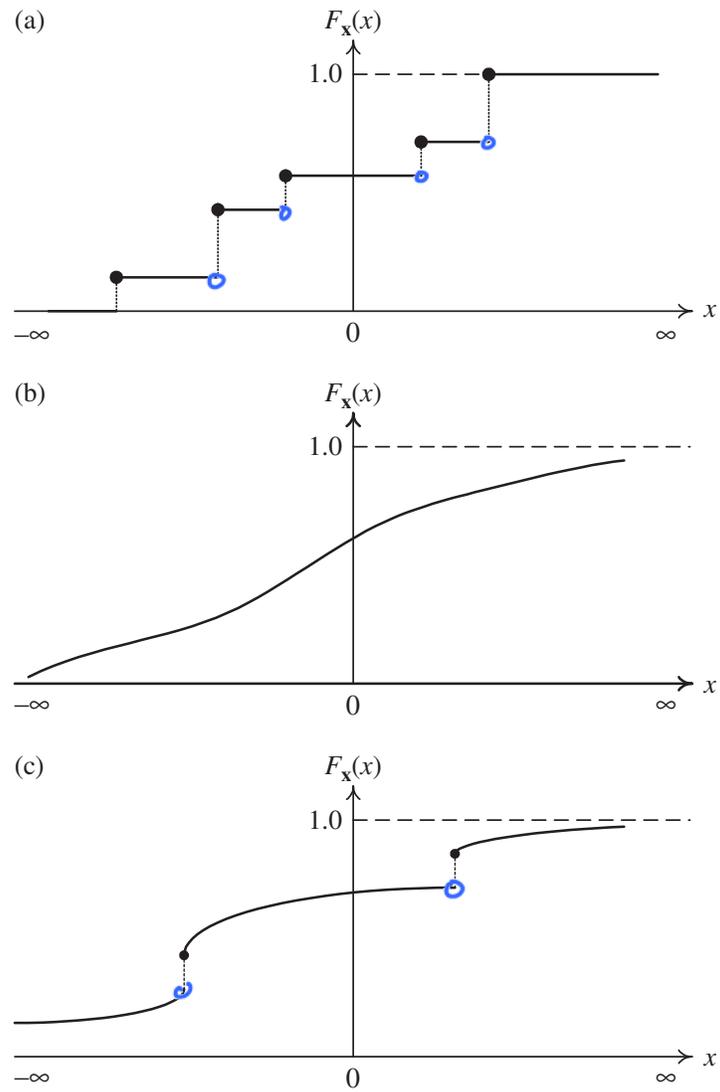


Figure 31: Typical cdfs: (a) a discrete random variable, (b) a continuous random variable, and (c) a mixed random variable [16, Fig. 3.2].

We have seen in Example 13.3 that some function can turn a continuous random variable into a mixed random variable. Next, we will work on an example where a continuous random variable is turned into a discrete random variable.

**Example 13.5.** Let  $X \sim \mathcal{U}(0, 1)$  and  $Y = g(X)$  where

$$g(x) = \begin{cases} 1, & x < 0.6 \\ 0, & x \geq 0.6. \end{cases}$$



Before going deeply into the math, it is helpful to think about the nature of the derived random variable  $Y$ . The definition of  $g(x)$  tells us that  $Y$  has only two possible values,  $Y = 0$  and  $Y = 1$ . Thus,  $Y$  is a discrete random variable.

$$P_Y(y) = \begin{cases} 0.4, & y=0, \\ 0.6, & y=1, \\ 0, & \text{otherwise.} \end{cases} \quad P[Y=0] = P[X \geq 0.6] = 0.4$$

**Example 13.6.** In MATLAB, we have the `rand` command to generate  $\mathcal{U}(0, 1)$ . If we want to generate a Bernoulli random variable with success probability  $p$ , what can we do?

$$Y = [\text{rand} < p]$$

**Exercise 13.7.** In MATLAB, how can we generate  $X \sim \text{binomial}(2, 1/4)$  from the `rand` command?