

Problem 1

a). $P[W \leq 4]$

$$F_X(x) = P[X \leq x]$$

$$F_W(4) = \frac{1}{4} + \frac{3(4-3)}{8} = \frac{1}{4} + \frac{3}{8} = \frac{5}{8}$$

b). $P[-2 < W \leq 2]$

$$P[a \leq X \leq b] = F_X(b) - F_X(a)$$

$$= \frac{1}{4} - \frac{1}{4} = 0 \quad \checkmark$$

c). $P[W > 0] = 1 - P[W \leq 0]$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

d). $P[W \leq a] = \frac{1}{2}$

$$a \in [3, 5)$$

$$F_W(a) = \frac{1}{2} = \frac{1}{4} + \frac{3(a-3)}{8}$$

$$a = \frac{11}{3}$$

Problem 2

a). $\int_{-\infty}^{\infty} f_X(x) dx = 1$

$$\int_0^2 cx dx = 1$$

$$c \int_0^2 x dx = 1$$

$$c \left. \frac{x^2}{2} \right|_0^2 = 1$$

$$2c - 0 = 1$$

$$c = \frac{1}{2}$$

b). $P[0 \leq X \leq 1] = \int_0^1 f_X(x) dx = \int_0^1 \frac{1}{2} x dx = \left. \frac{1}{2} \frac{x^2}{2} \right|_0^1 = \frac{1}{2} \cdot \frac{1}{2} - 0 = \frac{1}{4}$

c). $P[-\frac{1}{2} \leq X \leq \frac{1}{2}] = \int_{-\frac{1}{2}}^{\frac{1}{2}} f_X(x) dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{2} \frac{x^2}{2} dx = \frac{1}{2} \cdot \frac{1}{8} - \underbrace{0}_{[0, 2]} = \frac{1}{16}$