ECS 315: Probability and Random Processes **2014/1**

HW 6 — Due: Not Due

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Problem 1. The thickness of the wood paneling (in inches) that a customer orders is a random variable with the following cdf:

$$F_X(x) = \begin{cases} 0, & x < \frac{1}{8} \\ 0.2, & \frac{1}{8} \le x < \frac{1}{4} \\ 0.9, & \frac{1}{4} \le x < \frac{3}{8} \\ 1 & x \ge \frac{3}{8} \end{cases}$$

Determine the following probabilities:

(a) $P[X \le 1/18]$

(b) $P[X \le 1/4]$

(c) $P[X \le 5/16]$

(d) $P[X > 1/4]$

(e) $P[X \le 1/2]$

[Montgomery and Runger, 2010, Q3-42]

Problem 2. [M2011/1] The cdf of a random variable X is plotted in Figure 6.1.

(a) Find the pmf $p_X(x)$.

(b) Find the family to which X belongs. (Uniform, Bernoulli, Binomial, Geometric, Poisson, etc.)

Problem 3. Arrivals of customers at the local supermarket are modeled by a Poisson process with a rate of $\lambda = 2$ customers per minute. Let M be the number of customers arriving between 9:00 and 9:05. What is the probability that $M < 2$?

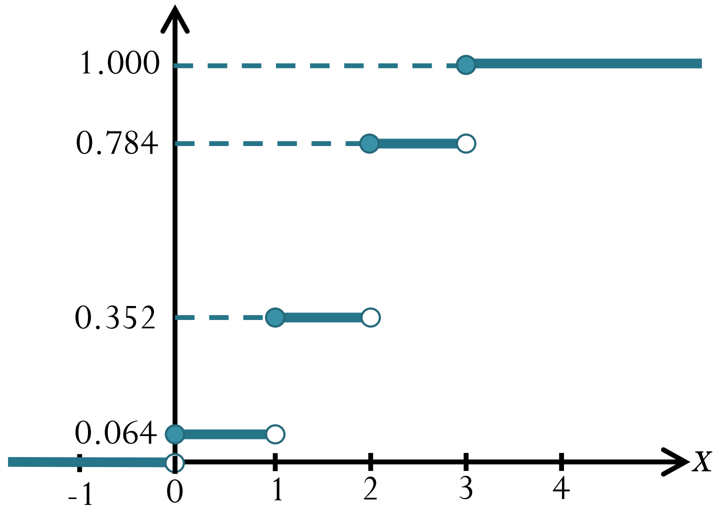

Figure 6.1: CDF of X for Problem 2

Problem 4. When n is large, binomial distribution Binomial(n, p) becomes difficult to compute directly because of the need to calculate factorial terms. In this question, we will consider an approximation when the value of p is close to 0. In such case, the binomial can be approximated[1] by the Poisson distribution with parameter $\alpha = np$. For this approximation to work, we will see in this exercise that n does not have to be very large and p does not need to be very small.

(a) Let $X \sim$ Binomial$(12, 1/36)$. (For example, roll two dice 12 times and let X be the number of times a double 6 appears.) Evaluate $p_X(x)$ for $x = 0, 1, 2$.

(b) Compare your answers part (a) with its Poisson approximation.

(c) Compare `MATLAB` plots of $p_X(x)$ in part (a) and the pmf of $\mathcal{P}(np)$.

Problem 5. You go to a party with 500 guests. What is the probability that exactly one other guest has the same birthday as you? Calculate this exactly and also approximately by using the Poisson pmf. (For simplicity, exclude birthdays on February 29.) [Bertsekas and Tsitsiklis, 2008, Q2.2.2]

[1] More specifically, suppose X_n has a binomial distribution with parameters n and p_n. If $p_n \to 0$ and $np_n \to \alpha$ as $n \to \infty$, then

$$P[X_n = k] \to e^{-\alpha} \frac{\alpha^k}{k!}.$$

Extra Questions

Here are some questions for those who want extra practice.

Problem 6. A sample of a radioactive material emits particles at a rate of 0.7 per second. Assuming that these are emitted in accordance with a Poisson distribution, find the probability that in one second

(a) exactly one is emitted,

(b) more than three are emitted,

(c) between one and four (inclusive) are emitted

[Applebaum, 2008, Q5.27].

Problem 7 (M2011/1)**.** You are given an unfair coin with probability of obtaining a head equal to $1/3,000,000,000$. You toss this coin 6,000,000,000 times. Let A be the event that you get "tails for all the tosses". Let B be the event that you get "heads for all the tosses".

(a) Approximate $P(A)$.

(b) Approximate $P(A \cup B)$.

Problem 8. In one of the New York state lottery games, a number is chosen at random between 0 and 999. Suppose you play this game 250 times. Use the Poisson approximation to estimate the probability that you will never win and compare this with the exact answer.