ECS 315: Probability and Random Processes **2014/1**

HW 5 — Due: Sep 25

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)
The extra question at the end is optional.

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1 (Majority Voting in Digital Communication)**.** A certain binary communication system has a bit-error rate of 0.1; i.e., in transmitting a single bit, the probability of receiving the bit in error is 0.1. To transmit messages, a three-bit repetition code is used. In other words, to send the message 1, a "codeword" 111 is transmitted, and to send the message 0, a "codeword" 000 is transmitted. At the receiver, if two or more 1s are received, the decoder decides that message 1 was sent; otherwise, i.e., if two or more zeros are received, it decides that message 0 was sent.

Assuming bit errors occur independently, find the probability that the decoder puts out the wrong message.

[Gubner, 2006, Q2.62]

Problem 2. An optical inspection system is to distinguish among different part types. The probability of a correct classification of any part is 0.98. Suppose that three parts are inspected and that the classifications are independent.

(a) Let the random variable X denote the number of parts that are correctly classified. Determine the probability mass function of X. [Montgomery and Runger, 2010, Q3-20]

(b) Let the random variable Y denote the number of parts that are incorrectly classified. Determine the probability mass function of Y.

Problem 3. Consider the sample space $\Omega = \{-2, -1, 0, 1, 2, 3, 4\}$. For an event $A \subset \Omega$, suppose that $P(A) = |A|/|\Omega|$. Define the random variable $X(\omega) = \omega^2$. Find the probability mass function of X.

Problem 4. Suppose X is a random variable whose pmf at $x = 0, 1, 2, 3, 4$ is given by $p_X(x) = \frac{2x+1}{25}$.

Remark: Note that the statement above does not specify the value of the $p_X(x)$ at the value of x that is not 0,1,2,3, or 4.

(a) What is $p_X(5)$?

(b) Determine the following probabilities:

  (i) $P[X = 4]$
  (ii) $P[X \le 1]$
  (iii) $P[2 \le X < 4]$
  (iv) $P[X > -10]$

Problem 5. The random variable V has pmf

$$p_V(v) = \begin{cases} cv^2, & v = 1, 2, 3, 4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant c.

(b) Find $P[V \in \{u^2 : u = 1, 2, 3, \ldots\}]$.

(c) Find the probability that V is an even number.

(d) Find $P[V > 2]$.

(e) Sketch $p_V(v)$.

(f) Sketch $F_V(v)$. (Note that $F_V(v) = P[V \le v]$.)

Extra Question

Problem 6. Consider a transmission over a binary symmetric channel (BSC) with crossover probability p. The random (binary) input to the BSC is denoted by X. Let p_1 be the probability that $X = 1$. (This implies the probability that $X = 0$ is $1 - p_1$.) Let Y by the output of the BSC.

(a) Suppose, at the receiver (which observes the output of the BSC), we learned that $Y = 1$. For each of the following scenarios, which event is more likely, "$X = 1$ was transmitted" or "$X = 0$ was transmitted"? (Hint: Use Bayes' theorem.)

  (i) Assume $p = 0.3$ and $p_1 = 0.1$.
  (ii) Assume $p = 0.3$ and $p_1 = 0.5$.
  (iii) Assume $p = 0.3$ and $p_1 = 0.9$.
  (iv) Assume $p = 0.7$ and $p_1 = 0.5$.

(b) Suppose, at the receiver (which observes the output of the BSC), we learned that $Y = 0$. For each of the following scenarios, which event is more likely, "$X = 1$ was transmitted" or "$X = 0$ was transmitted"?

  (i) Assume $p = 0.3$ and $p_1 = 0.1$
  (ii) Assume $p = 0.3$ and $p_1 = 0.5$
  (iii) Assume $p = 0.3$ and $p_1 = 0.9$
  (iv) Assume $p = 0.7$ and $p_1 = 0.5$

Remark: A MAP (maximum a posteriori) detector is a detector that takes the observed value Y and then calculate the most likely transmitted value. More specifically,

$$\hat{x}_{MAP}(y) = \arg\max_x P\left[X = x | Y = y\right]$$

In fact, in part (a), each of your answers is $\hat{x}_{MAP}(1)$ and in part (b), each of your answers is $\hat{x}_{MAP}(0)$.