ECS 315: Probability and Random Processes	2014/1

HW 4 — Due: September 18

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Let A denote the event that the design color is red and let B denote the event that the font size is not the smallest one.

(a) Use classical probability to evaluate $P(A)$, $P(B)$ and $P(A \cap B)$. Show that the two events A and B are independent by checking whether $P(A \cap B) = P(A)P(B)$.

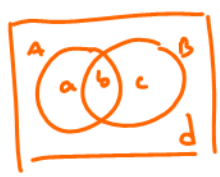
(b) Using the values of $P(A)$ and $P(B)$ from the previous part and the fact that $A \perp\!\!\!\perp B$, calculate the following probabilities.

$P(A \cap B) = P(A)\,P(B)$

(i) $P(A \cup B)$

(ii) $P(A \cup B^c)$

(iii) $P(A^c \cup B^c)$



$P(A \cap B^c) = P(A)P(B^c)$

[Montgomery and Runger, 2010, Q2-84]

Problem 2. In this question, each experiment has equiprobable outcomes.

(a) Let $\Omega = \{1, 2, 3, 4\}$, $A_1 = \{1, 2\}$, $A_2 = \{1, 3\}$, $A_3 = \{2, 3\}$.

(i) Determine whether $P(A_i \cap A_j) = P(A_i)\,P(A_j)$ for all $i \neq j$.

(ii) Check whether $P(A_1 \cap A_2 \cap A_3) = P(A_1) P(A_2) P(A_3)$.

(iii) Are A_1, A_2, and A_3 independent?

(b) Let $\Omega = \{1, 2, 3, 4, 5, 6\}$, $A_1 = \{1, 2, 3, 4\}$, $A_2 = A_3 = \{4, 5, 6\}$.

(i) Check whether $P(A_1 \cap A_2 \cap A_3) = P(A_1) P(A_2) P(A_3)$.

(ii) Check whether $P(A_i \cap A_j) = P(A_i) P(A_j)$ for all $i \neq j$.

(iii) Are A_1, A_2, and A_3 independent?

Problem 3. In an experiment, A, B, C, and D are events with probabilities $P(A \cup B) = \frac{5}{8}$, $P(A) = \frac{3}{8}$, $P(C \cap D) = \frac{1}{3}$, and $P(C) = \frac{1}{2}$. Furthermore, A and B are disjoint, while C and D are independent.

(a) Find

(i) $P(A \cap B)$

(ii) $P(B)$

(iii) $P(A \cap B^c)$

(iv) $P(A \cup B^c)$

(b) Are A and B independent?

(c) Find

(i) $P(D)$

(ii) $P(C \cap D^c)$

(iii) $P(C^c \cap D^c)$

(iv) $P(C|D)$

(v) $P(C \cup D)$

(vi) $P(C \cup D^c)$

(d) Are C and D^c independent?

Problem 4. Series Circuit: The circuit in Figure 4.1 operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates? [Montgomery and Runger, 2010, Ex. 2-32]

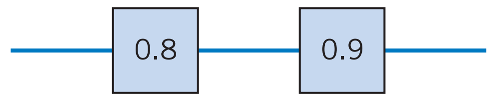
L = event that the device on the LHS functions properly.

R = " RHS "

$L^c \perp\!\!\!\perp R^c$

$\Updownarrow$

$L \perp\!\!\!\perp R$



$P(L \cap R) = P(L)P(R)$

Figure 4.1: Circuit for Problem 4

Extra Questions

Here are optional questions for those who want more practice.

Problem 5. Show that if A and B are independent events, then so are A and B^c, A^c and B, and A^c and B^c.

$P(A \cup B) = 1 - P(A^c \cap B^c)$

Problem 6. Anne and Betty go fishing. Find the conditional probability that Anne catches no fish given that at least one of them catches no fish. Assume they catch fish independently and that each has probability $0 < p < 1$ of catching no fish. [Gubner, 2006, Q2.62]

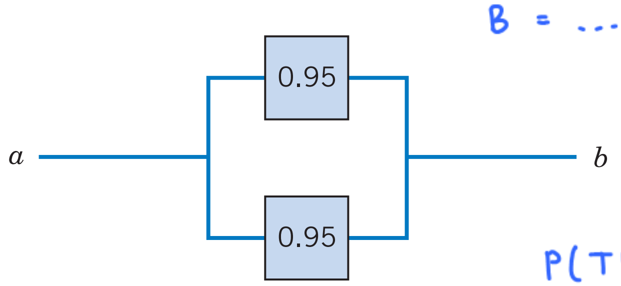
Hint: Let A be the event that Anne catches no fish and B be the event that Betty catches no fish. Observe that the question asks you to evaluate $P(A|(A \cup B))$.

$$P(A|(A \cup B)) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{p}{P(A) + P(B) - P(A \cap B)}$$

Problem 7. The circuit in Figure 4.2 operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates? [Montgomery and Runger, 2010, Ex. 2-34]

$T = \ldots$

$B = \ldots.$



$$P(T \cup B) \overset{1}{=} P(T) + P(B) - P(T \cap B)$$

$(A_1 \cup A_2 \cup \cdots \cup A_n)^c$

Figure 4.2: Circuit for Problem 7

$$= 1 - P((T \cup B)^c)$$

2

$$= 1 - P(T^c \cap B^c)$$

$$= 1 - P(T^c)P(B^c)$$

$$= 1 - (0.05)^2$$

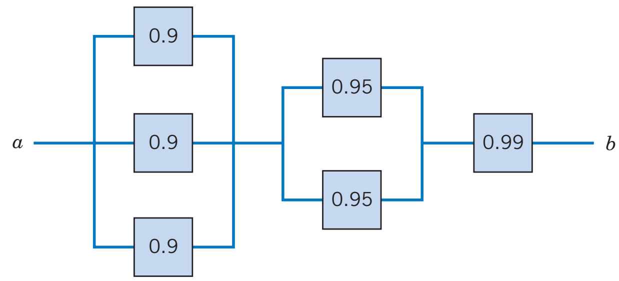


Figure 4.3: Circuit for Problem 8

Problem 8. The circuit in Figure 4.3 operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates? [Montgomery and Runger, 2010, Ex. 2-35]

Problem 9. Show that

(a) $P(A \cap B \cap C) = P(A) \times P(B|A) \times P(C|A \cap B)$.

(b) $P(B \cap C|A) = P(B|A)P(C|B \cap A)$