ECS 315: Probability and Random Processes **2014/1**

HW 13 — Due: Not Due

Lecturer: Prapun Suksompong, Ph.D.

Problem 1. Complete the table below. Make sure that you use the definitions/notations that are presented in class.

$X \sim$	Support (S_X)	pmf/pdf on S_X	$\mathbb{E}X$	$\operatorname{Var} X$
$\mathcal{U}(\{n, n+1, \dots, n+d\})$				
Bernoulli(p)	$\{0, 1\}$	$\begin{cases} 1-p, & x = 0, \\ p, & x = 1. \end{cases}$	p	$p(1-p)$
Binomial(n, p)				
$\mathcal{G}_0(\beta)$	$\{0, 1, 2, \dots\}$			
$\mathcal{G}_1(\beta)$	$\mathbb{N}$		$\frac{1}{1-\beta}$	
$\mathcal{P}(\alpha)$				
$\mathcal{U}(a, b)$				
$\mathcal{E}(\lambda)$		$\lambda e^{-\lambda x}$		
$\mathcal{N}(m, \sigma^2)$	$(-\infty, +\infty)$	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$	m	σ^2

Problem 2. In this question, we will explore the relationship between exponential random variable and geometric random variable.

(a) Start with an exponential random variable X whose parameter is λ. What is its pdf?

(b) What is the probability that X is in the interval $[a, b)$ for constants $b > a \geq 0$?

(c) What is the probability that X is in the interval $I_k = [(k-1)T, kT)$? Assume T is a positive real number and k is a positive integer. We will denote this probability by $p_k = P[X \in I_k]$.

(d) Consider the sequence of numbers $p_1, p_2, p_3, \dots$ where $p_k = P[X \in I_k]$ defined above. Are these p_k's agrees with a pmf of a geometric random variable? If so, what is the value of the parameter β for this geometric random variable?

(e) Let $Y = \lfloor X \rfloor$ where $\lfloor \cdot \rfloor$ is the floor function. Describe the random variable Y. (Continuous or discrete? What is its pdf/pmf?)

Problem 3. Consider the function

$$g(x) = \begin{cases} x, & x \geq 0 \\ 0, & x < 0. \end{cases}$$

The function g operates like a **half-wave rectifier** in that if a positive voltage x is applied, the output is $y = x$, while if a negative voltage x is applied, the output is $y = 0$. Suppose $Y = g(X)$, where $X \sim \mathcal{U}(-1, 1)$. Plot the cdf of Y.

Problem 4. Suppose the elements of a $1 \times n$ random vector $\mathbf{A}$ and an $n \times n$ random matrix $\mathbf{B}$ are all i.i.d. random variables with shared pmf

$$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, 3, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Let $C = \frac{1}{n}\left(\mathbf{A} \times \mathbf{A}^T\right)$. Use the law of large numbers to estimate the value of C when n is large.

(b) Let $\mathbf{D} = \frac{1}{n}\left(\mathbf{B} \times \mathbf{B}^T\right)$. Use the law of large numbers to estimate the values of the elements inside matrix $\mathbf{D}$ when n is large.

Problem 5. A student has passed a final exam by supplying correct answers for 26 out of 50 multiple-choice questions. For each question, there was a choice of three possible answers, of which only one was correct. The student claims not to have learned anything in the course and not to have studied for the exam, and says that his correct answers are the product of guesswork. Use Table 3.1 and/or Table 3.2 from [Yates & Goodman, 2005] to determine whether you should believe him.