ECS 315: Probability and Random Processes **2014/1**

HW 12 — Due: Dec 4

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)
The extra questions at the end are optional.

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. The input X and output Y of a system subject to random perturbations are described probabilistically by the following joint pmf matrix:

x \ y	2	4	5
1	0.02	0.10	0.08
3	0.08	0.32	0.40

Evaluate the following quantities:

(a) The marginal pmf $p_X(x)$

(b) The marginal pmf $p_Y(y)$

(c) $\mathbb{E}X$

(d) $\operatorname{Var} X$

(e) $\mathbb{E}Y$

(f) $\operatorname{Var} Y$

(g) $P[XY < 6]$

(h) $P[X = Y]$

Problem 2. The input X and output Y of a system subject to random perturbations are described probabilistically by the following joint pmf matrix:

P_{XY}

x \ y	2	4	5
1	0.02	0.10	0.08
3	0.08	0.32	0.40

meshgrid(x, y)

$X \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$

$Y \begin{bmatrix} 2 & 4 & 5 \\ 2 & 4 & 5 \end{bmatrix}$

(a) Evaluate the following quantities:

(i) $\mathbb{E}[XY]$

(ii) $\mathbb{E}[(X-3)(Y-2)]$

(iii) $\mathbb{E}[X(Y^3 - 11Y^2 + 38Y)]$

(iv) $\mathrm{Cov}[X, Y]$

(v) $\rho_{X,Y}$

sum(sum(X.*Y.*P_{XY}))

$\mathbb{E}[(X+Y)^2]$

Hint: Write down the formulas then use `MATLAB` or `Excel` to compute them.

(b) Find $\rho_{X,X}$

(c) Calculate the following quantities using the values of $\mathrm{Var}\,X$, $\mathrm{Cov}[X, Y]$, and $\rho_{X,Y}$ that you got earlier.

$\mathrm{Cov}[X, Y] = \mathbb{E}[(X - \mathbb{E}X)(Y - \mathbb{E}Y)]$

(i) $\mathrm{Cov}[3X + 4, 6Y - 7] = 3 \times 6 \times \mathrm{Cov}[X, Y]$

(ii) $\rho_{3X+4,6Y-7}$

(iii) $\mathrm{Cov}[X, 6X - 7]$

(iv) $\rho_{X,6X-7}$

Problem 3. Suppose $X \sim \text{binomial}(5, 1/3)$, $Y \sim \text{binomial}(7, 4/5)$, and $X \perp\!\!\!\perp Y$. Evaluate the following quantities.

(a) $\mathbb{E}[(X-3)(Y-2)]$

(b) $\mathrm{Cov}[X,Y]$

(c) $\rho_{X,Y}$

Problem 4. Suppose we know that $\sigma_X = \frac{\sqrt{21}}{10}$, $\sigma_Y = \frac{4\sqrt{6}}{5}$, $\rho_{X,Y} = -\frac{1}{\sqrt{126}}$.

(a) Find $\mathrm{Var}[X+Y]$.

(b) Find $\mathbb{E}[(Y-3X+5)^2]$. Assume $\mathbb{E}[Y-3X+5] = 1$.

Extra Question

Here are optional questions for those who want extra practice.

Problem 5. The input X and output Y of a system subject to random perturbations are described probabilistically by the joint pmf $p_{X,Y}(x,y)$, where $x = 1,2,3$ and $y = 1,2,3,4,5$. Let P denote the joint pmf matrix whose i,j entry is $p_{X,Y}(i,j)$, and suppose that

$$P = \frac{1}{71}\begin{bmatrix} 7 & 2 & 8 & 5 & 4 \\ 4 & 2 & 5 & 5 & 9 \\ 2 & 4 & 8 & 5 & 1 \end{bmatrix}$$

(a) Find the marginal pmfs $p_X(x)$ and $p_Y(y)$.

(b) Find $\mathbb{E}X$

(c) Find $\mathbb{E}Y$

(d) Find $\mathrm{Var}\,X$

(e) Find $\mathrm{Var}\,Y$

Problem 6. A webpage server can handle r requests per day. Find the probability that the server gets more than r requests at least once in n days. Assume that the number of requests on day i is $X_i \sim \mathcal{P}(\alpha)$ and that $X_1, \ldots, X_n$ are independent.

Problem 7. Suppose $X \sim \mathrm{binomial}(5, 1/3)$, $Y \sim \mathrm{binomial}(7, 4/5)$, and $X \perp\!\!\!\perp Y$.

(a) A vector describing the pmf of X can be created by the MATLAB expression:

```
x = 0:5; pX = binopdf(x,5,1/3).
```

What is the expression that would give `pY`, a corresponding vector describing the pmf of Y?

(b) Use `pX` and `pY` from part (a), how can you create the joint pmf matrix in MATLAB? Do not use "for-loop", "while-loop", "if statement". Hint: Multiply them in an appropriate orientation.

(c) Use `MATLAB` to evaluate the following quantities. Again, do not use "for-loop", "while-loop", "if statement".

(i) $\mathbb{E}X$

(ii) $P[X = Y]$

(iii) $P[XY < 6]$

Problem 8. Suppose $\operatorname{Var} X = 5$. Find $\operatorname{Cov}[X, X]$ and $\rho_{X,X}$.