ECS315 Quiz 1 Solution

Tuesday, September 2, 2014 7:44 PM

Data from fifteen testees are shown below.

D : 0 1 1 0 0 0 0 1 1 1 1 0 1 0 1

T_p : 1 0 0 1 1 0 0 0 0 0 1 1 0 1 1

$P(D \cap T_p) = \frac{2}{15}$

$P(D^c \cap T_p) = \frac{5}{15}$

(Assume the same definitions as those defined in class.)

$P(T_p | D) = \frac{P(T_p \cap D)}{P(D)} = \frac{2/15}{8/15} = \frac{2}{8} = \frac{1}{4}$

Use the provided data to estimate the following probabilities:

$P(D | T_p) = \frac{2}{7}$

Among the 15 testees, 8 of them have the disease

$P(D) \approx \boxed{\frac{8}{15}}$

$P(D^c) \approx \boxed{\frac{7}{15}}$

Among the 8 testees who have the disease, two are tested positive

$P(T_p | D) \approx \frac{2}{8} = \boxed{\frac{1}{4}}$

$P(T_p | D^c) \approx \boxed{\frac{5}{7}}$

$P(T_p^c | D) \approx \frac{6}{8} = \boxed{\frac{3}{4}}$

$P(T_p^c | D^c) \approx \boxed{\frac{2}{7}}$

$P(T_p) \approx \boxed{\frac{7}{15}}$

$P(T_p^c) \approx \boxed{\frac{8}{15}}$

Note that ① $P(T_p) = P(T_p|D)P(D) + P(T_p|D^c)P(D^c)$

total probability theorem

② $P(D|T_p) = \frac{P(T_p|D)P(D)}{P(T_p)}$

Bayes' theorem

ECS315 Quiz 2 Sol

Tuesday, September 16, 2014 2:03 PM

Consider a random experiment in which you roll a 20-sided fair dice.

$(\Omega = \{1, 2, 3, \dots, 20\})$

Let $X(\omega) = \omega$ $\quad$ $Z(\omega) = |\omega - 5| - 3$

$Y(\omega) = (\omega - 5)^2$

① Find $P[X = 5]$

$X(\omega) = 5$ iff $\omega = 5$

So, $P[X=5] = P(\{5\}) = \frac{1}{20}$

② Find $P[Y = 16]$

$Y(\omega) = 16$ iff $(\omega - 5)^2 = 16$

$\omega - 5 = \pm 4$

$\omega = 5 \pm 4 = 1$ or 9

So, $P[Y = 16] = P(\{1, 9\}) = \frac{2}{20} = \frac{1}{10}$

③ Find $P[Y > 10]$

$Y(\omega) > 10$ iff $(\omega - 5)^2 > 10$

Here, we plug-in $\omega = 1, 2, \dots, 20$ one-by-one and see that $\omega = 1, 9, 10, 11, \dots, 20$ satisfy the condition.

ω	$(\omega-5)^2$	>10?
1	16	✓
2	9	×
3	4	×
4	1	×
5	0	×
6	1	×
7	4	×
8	9	×
9	16	✓
10	25	✓
⋮	↓ increasing so, >10	⋮

So, $P[Y > 10] = P(\{1, 9, 10, 11, \dots, 20\}) = \frac{13}{20}$

Alternatively, you may remember that $(\omega-5)^2 > 10$ iff $\omega - 5 > \sqrt{10}$ or $\omega - 5 < -\sqrt{10}$

$\Updownarrow$ $\quad$ $\Updownarrow$

$\omega > \underbrace{5 + \sqrt{10}}_{= 8.1623}$ $\quad$ $\omega < \underbrace{5 - \sqrt{10}}_{1.8377}$

$\Updownarrow$ $\quad$ $\Updownarrow$

$\omega = 9, 10, \dots, 20$ $\quad$ $\omega = 1$

④ Find $P[Z > 10]$

$Z(\omega) > 10$ iff $|\omega - 5| - 3 > 10$

$|\omega - 5| > 13$

Here, we plug-in $\omega = 1, 2, \dots, 20$ one-by-one and see that $\omega = 19, 20$ satisfy the condition.

ω	$\lvert\omega-5\rvert$	>13?	$8 < \cdot < 13$?
1	4	×	×
2	3	×	×
3	2	×	×
4	1	×	×
5	0	×	×
6	1	×	×
7	2	×	×
8	3	×	×
9	4	×	×
10	5	×	×
11	6	×	×
12	7	×	×

So $P[Z > 10] = P(\{19, 20\}) = \frac{2}{20} = \frac{1}{10}$

8	3	×	×
9	4	×	×
10	5	×	×
11	6	×	×
12	7	×	×
13	8	×	×
14	9	×	✓
15	10	×	✓
16	11	×	✓
17	12	×	✓
18	13	×	×
19	14	✓	×
20	15	✓	×

condition.

So, $P[Z > 10] = P(\{19, 20\}) = \frac{2}{20} = \frac{1}{10}$

Alternatively, you may remember that $|\omega - 5| > 13$ iff $\omega - 5 > 13$ or $\omega - 5 < -13$

$\omega - 5 > 13 \Leftrightarrow \omega > 18 \Leftrightarrow \omega = 19, 20$

$\omega - 5 < -13 \Leftrightarrow \omega < -8$ ← No ω satisfies this

⑤ Find $P[5 < Z < 10]$

$5 < Z(\omega) < 10$ iff $5 < |\omega - 5| - 3 < 10$

$8 < |\omega - 5| < 13$

Here, we plug-in $\omega = 1, 3, \ldots, 20$ one-by-one and see that $\omega = 14, 15, 16, 17$ satisfy the condition.

So, $P[5 < Z < 10] = P(\{14, 15, 16, 17\}) = \frac{4}{20} = \frac{1}{5}$

Alternatively, you may remember that $|\omega - 5| > 8$ iff $\omega - 5 > 8$ or $\omega - 5 < -8$

$\omega - 5 > 8 \Leftrightarrow \omega > 13$

$\omega - 5 < -8 \Leftrightarrow \omega < -3$ ← No ω satisfies this

$|\omega - 5| < 13$ iff $-13 < \omega - 5 < 13 \Leftrightarrow -8 < \omega < 18$

Here, ω need to satisfy both these condition. So, $13 < \omega < 18 \Leftrightarrow \omega = 14, 15, 16, 17$

ECS315 Quiz 3 Solution

Friday, September 19, 2014 10:01 AM

Suppose a RV X has pmf $p_X(x) = \begin{cases} c/x, & x = 1,2,3 \\ 0, & \text{otherwise.} \end{cases}$

Sketch $P[X \le x]$ for $x \in \mathbb{R}$.

In lecture, we found that $p_X(x) = \begin{cases} 6/11, & x = 1, \\ 3/11, & x = 2, \\ 2/11, & x = 3, \\ 0, & \text{otherwise.} \end{cases}$

Note first that the RV X can only be 1, 2, or 3.

We may start by evaluting $P[X \le x]$ at many values of x. We then realize the following:

When $x < 1$, $P[X \le x] = 0$ because the possible values for RV X are all ≥ 1

When $1 \le x < 2$, $P[X \le x] = \frac{6}{11}$ because only the event "$X=1$" satisfies the condition "$X \le x$".

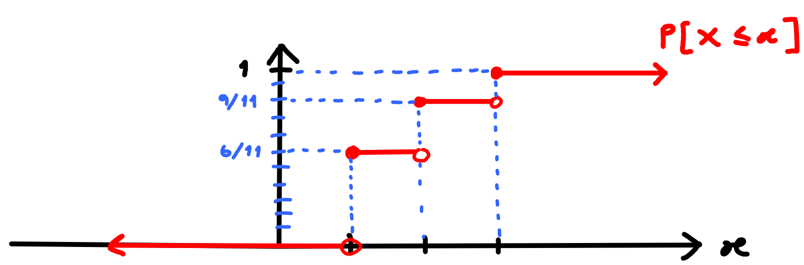
When $2 \le x < 3$, $P[X \le x] = \frac{6}{11} + \frac{3}{11} = \frac{9}{11}$ because exactly two events "$X=1$" and "$X=2$" satisfy the condition "$X \le x$".

When $x \geq 3$, $P[X \le x] = 1$ because all possible values for RV X are now $\le x$.

So,

$$P[X \le x] = \begin{cases} 0, & x < 1, \\ 6/11, & 1 \le x < 2, \\ 9/11, & 2 \le x < 3, \\ 1, & x \geq 3. \end{cases}$$

Here is a sketch of $P[X \le x]$



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