ECS 315: Probability and Random Processes **2013/1**

HW 8 — Due: Sep 6

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

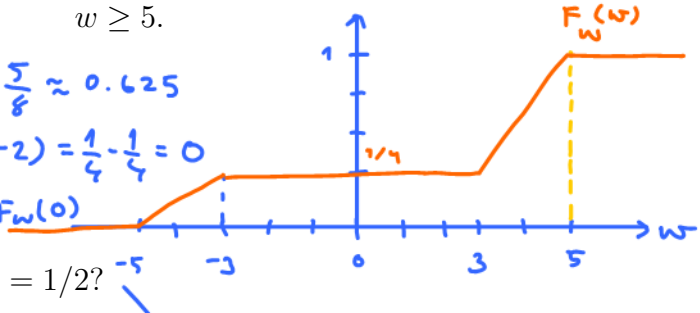
(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1 (Yates and Goodman, 2005, Q3.1.3)**.** The CDF of random variable W is

$$P[W \le w] = F_W(w) = \begin{cases} 0, & w < -5, \\ (w+5)/8, & -5 \le w < -3, \\ 1/4, & -3 \le w < 3, \\ 1/4 + 3(w-3)/8, & 3 \le w < 5, \\ 1, & w \ge 5. \end{cases}$$



(a) What is $P[W \le 4]$? $= F_W(4) = \frac{1}{4} + 3 \times \frac{1}{8} = \frac{5}{8} \approx 0.625$

(b) What is $P[-2 < W \le 2]$? $= F_W(2) - F_W(-2) = \frac{1}{4} - \frac{1}{4} = 0$

(c) What is $P[W > 0]$? $= 1 - P[W \le 0] = 1 - F_W(0) = 3/4$

(d) What is the value of a such that $P[W \le a] = 1/2$? $\frac{1}{4} + \frac{2}{8}(a-3) = \frac{1}{2} \Rightarrow a = \frac{11}{3}$ median

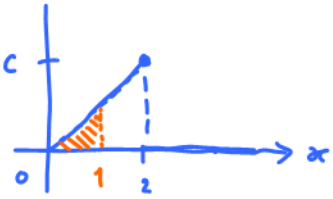
Problem 2 (Yates and Goodman, 2005, Q3.2.1)**.** The random variable X has probability density function

$$f_X(x) = \begin{cases} cx & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Use the pdf to find

(a) the constant c, $\int_{-\infty}^{\infty} f_x(x)\,dx = \int_0^2 cx\,dx = 1 \Rightarrow c = \frac{1}{2}$

(b) $P[0 \le X \le 1]$, $\int_0^1 cx\,dx = c\frac{x^2}{2}\Big|_0^1 = \frac{1}{4}$ $\approx \frac{1}{4}$



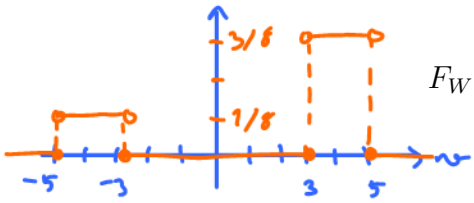
$\frac{1}{2} \times 2 \times 2c = 1$

$c = 1/2$

(c) $P[-1/2 \le X \le 1/2]$,

(d) the cdf $F_X(x)$.

Problem 3 (Yates and Goodman, 2005, Q3.2.3). The CDF of random variable W is



$$F_W(w) = \begin{cases} 0, & w < -5, \\ (w+5)/8, & -5 \le w < -3, \\ 1/4, & -3 \le w < 3, \\ 1/4 + 3(w-3)/8, & 3 \le w < 5, \\ 1, & w \ge 5. \end{cases}$$

Find its pdf $f_W(w)$.

Problem 4 (Yates and Goodman, 2005, Q3.3.4). The pdf of random variable Y is

$$f_Y(y) = \begin{cases} y/2 & 0 \le y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

What are $\mathbb{E}[Y]$ and $\mathrm{Var}\,Y$?

Problem 5 (Yates and Goodman, 2005, Q3.3.6). The cdf of random variable V is

$$F_V(v) = \begin{cases} 0 & v < -5, \\ (v+5)^2/144, & -5 \le v < 7, \\ 1 & v \ge 7. \end{cases}$$

(a) What is $\mathbb{E}[V]$?

(b) What is $\mathrm{Var}[V]$?

(c) What is $\mathbb{E}[V^3]$?