ECS 315: Probability and Random Processes **2016/1**

HW 6 — Due: Not Due

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Problem 1. The thickness of the wood paneling (in inches) that a customer orders is a random variable with the following cdf:

$$F_X(x) = \begin{cases} 0, & x < \frac{1}{8} \\ 0.2, & \frac{1}{8} \le x < \frac{1}{4} \\ 0.9, & \frac{1}{4} \le x < \frac{3}{8} \\ 1 & x \ge \frac{3}{8} \end{cases}$$

Determine the following probabilities:

(a) $P[X \le 1/18]$

(b) $P[X \le 1/4]$

(c) $P[X \le 5/16]$

(d) $P[X > 1/4]$

(e) $P[X \le 1/2]$

[Montgomery and Runger, 2010, Q3-42]

Problem 2. [M2011/1] The cdf of a random variable X is plotted in Figure 6.1.

(a) Find the pmf $p_X(x)$.

(b) Find the family to which X belongs? (Uniform, Bernoulli, Binomial, Geometric, Poisson, etc.)

Extra Questions

Here are some questions for those who want extra practice.

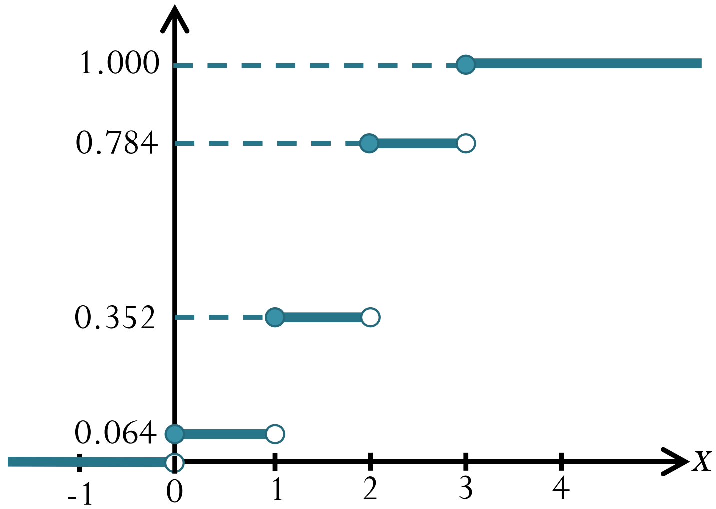

Figure 6.1: CDF of X for Problem 2

Problem 3. When n is large, binomial distribution Binomial(n, p) becomes difficult to compute directly because of the need to calculate factorial terms. In this question, we will consider an approximation when p is close to 0. In such case, the binomial can be approximated[1] by the Poisson distribution with parameter $\alpha = np$.

(a) Let $X \sim$ Binomial$(12, 1/36)$. (For example, roll two dice 12 times and let X be the number of times a double 6 appears.) Evaluate $p_X(x)$ for $x = 0, 1, 2$.

(b) Compare your answers in the previous part with the Poisson approximation.

(c) Compare `MATLAB` plots of $p_X(x)$ and the pmf of $\mathcal{P}(np)$.

(d) In one of the New York state lottery games, a number is chosen at random between 0 and 999. Suppose you play this game 250 times. Use the Poisson approximation to estimate the probability that you will never win and compare this with the exact answer.

Problem 4. For each of the following families of random variable X, find the value(s) of x which maximize $p_X(x)$. (This can be interpreted as the "mode" of X.)

[1] More specifically, suppose X_n has a binomial distribution with parameters n and p_n. If $p_n \to 0$ and $np_n \to \alpha$ as $n \to \infty$, then

$$P[X_n = k] \to e^{-\alpha} \frac{\alpha^k}{k!}.$$

(a) $\mathcal{P}(\alpha)$

(b) Binomial(n, p)

(c) $\mathcal{G}_0(\beta)$

(d) $\mathcal{G}_1(\beta)$

Remark [Y&G, p. 66]:

- For statisticians, the mode is the most common number in the collection of observations. There are as many or more numbers with that value than any other value. If there are two or more numbers with this property, the collection of observations is called multimodal. In probability theory, a **mode** of random variable X is a number x_{mode} satisfying
$$p_X(x_{\text{mode}}) \geq p_X(x) \quad \text{for all } x.$$

- For statisticians, the median is a number in the middle of the set of numbers, in the sense that an equal number of members of the set are below the median and above the median. In probability theory, a median, X_{median}, of random variable X is a number that satisfies
$$P[X < X_{\text{median}}] = P[X > X_{\text{median}}].$$

- Neither the mode nor the median of a random variable X need be unique. A random variable can have several modes or medians.