

HW 5 — Due: Sep 26

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Instructions

- (a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.
- (b) It is important that you try to solve all problems. (5 pt)
- (c) Late submission will be heavily penalized.
- (d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.



Problem 1 (Majority Voting in Digital Communication). A certain binary communication system has a bit-error rate of 0.1; i.e., in transmitting a single bit, the probability of receiving the bit in error is 0.1. To transmit messages, a three-bit repetition code is used. In other words, to send the message 1, a “codeword” 111 is transmitted, and to send the message 0, a “codeword” 000 is transmitted. At the receiver, if two or more 1s are received, the decoder decides that message 1 was sent; otherwise, i.e., if two or more zeros are received, it decides that message 0 was sent.

Assuming bit errors occur independently, find the probability that the decoder puts out the wrong message.

[Gubner, 2006, Q2.62]

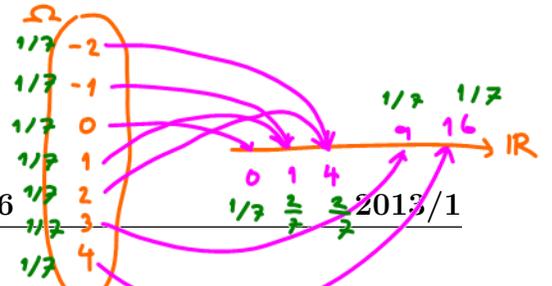
(1) Without repetition code, $P(\mathcal{E}) = p$.

(2) With repetition code,

$$P(\mathcal{E}) = P[\# \text{ bits flipped} = 2, 3] = \binom{3}{2} p^2 (1-p) + p^3 = \dots$$

Problem 2. An optical inspection system is to distinguish among different part types. The probability of a correct classification of any part is 0.98. Suppose that three parts are inspected and that the classifications are independent.

- (a) Let the random variable X denote the number of parts that are correctly classified. Determine the probability mass function of X . [Montgomery and Runger, 2010, Q3-20]
- (b) Let the random variable Y denote the number of parts that are incorrectly classified. Determine the probability mass function of Y .



Problem 3. Consider the sample space $\Omega = \{-2, -1, 0, 1, 2, 3, 4\}$. For an event $A \subset \Omega$, suppose that $P(A) = |A|/|\Omega|$. Define the random variable $X(\omega) = \omega^2$. Find the probability mass function of X .

$$P_X(x) = \begin{cases} 1/7, & x = 0, 9, 16, \\ 2/7, & x = 1, 4, \\ 0, & \text{otherwise.} \end{cases}$$

Problem 4. Suppose X is a random variable whose pmf at $x = 0, 1, 2, 3, 4$ is given by $p_X(x) = \frac{2x+1}{25}$.

Remark: Note that the statement above does not specify the value of the $p_X(x)$ at the value of x that is not 0,1,2,3, or 4.

- (a) What is $p_X(5)$?
- (b) Determine the following probabilities:
 - (i) $P[X = 4]$
 - (ii) $P[X \leq 1]$
 - (iii) $P[2 \leq X < 4]$
 - (iv) $P[X > -10]$

Problem 5. The random variable V has pmf

$$p_V(v) = \begin{cases} cv^2, & v = 1, 2, 3, 4, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the value of the constant c .
- (b) Find $P[V \in \{u^2 : u = 1, 2, 3, \dots\}]$.
- (c) Find the probability that V is an even number.
- (d) Find $P[V > 2]$.
- (e) Sketch $p_V(v)$.
- (f) Sketch $F_V(v)$.