ECS 315: Probability and Random Processes **2013/1**

HW 4 — Due: July 19

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Show that if A and B are independent events, then so are A^c and B^c.

An extra problem at the end of this assignment considers the other cases.

Problem 2. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Let A denote the event that the design color is red and let B denote the event that the font size is not the smallest one.

(a) Use classical probability to evaluate $P(A)$, $P(B)$ and $P(A \cap B)$. Show that the two events A and B are independent by checking whether $P(A \cap B) = P(A)P(B)$.

(b) Using the values of $P(A)$ and $P(B)$ from the previous part and the fact that $A \perp\!\!\!\perp B$, calculate the following probabilities.

(i) $P(A \cup B)$

(ii) $P(A \cup B^c)$

(iii) $P(A^c \cup B^c)$

[Montgomery and Runger, 2010, Q2-84]

Problem 3. In this question, each experiment has equiprobable outcomes.

(a) Let $\Omega = \{1,2,3,4\}$, $A_1 = \{1,2\}$, $A_2 = \{1,3\}$, $A_3 = \{2,3\}$.

(i) Determine whether $P(A_i \cap A_j) = P(A_i)P(A_j)$ for all $i \neq j$.

(ii) Check whether $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$.

(iii) Are A_1, A_2, and A_3 independent?

(b) Let $\Omega = \{1,2,3,4,5,6\}$, $A_1 = \{1,2,3,4\}$, $A_2 = A_3 = \{4,5,6\}$.

(i) Check whether $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$.

(ii) Check whether $P(A_i \cap A_j) = P(A_i)P(A_j)$ for all $i \neq j$.

(iii) Are A_1, A_2, and A_3 independent?

Problem 4. In an experiment, A, B, C, and D are events with probabilities $P(A \cup B) = \frac{5}{8}$, $P(A) = \frac{3}{8}$, $P(C \cap D) = \frac{1}{3}$, and $P(C) = \frac{1}{2}$. Furthermore, A and B are disjoint, while C and D are independent.

(a) Find

(i) $P(A \cap B)$

(ii) $P(B)$

(iii) $P(A \cap B^c)$

(iv) $P(A \cup B^c)$

(b) Are A and B independent?

(c) Find

(i) $P(D)$

(ii) $P(C \cap D^c)$

(iii) $P(C^c \cap D^c)$

(iv) $P(C|D)$

(v) $P(C \cup D)$

(vi) $P(C \cup D^c)$

(d) Are C and D^c independent?

Extra Questions

Here are optional questions for those who want more practice.

Problem 5. Show that if A and B are independent events, then so are A and B^c, A^c and B, and A^c and B^c.

Problem 6. Anne and Betty go fishing. Find the conditional probability that Anne catches no fish given that at least one of them catches no fish. Assume they catch fish independently and that each has probability $0 < p < 1$ of catching no fish. [Gubner, 2006, Q2.62]

Problem 7. Show that

(a) $P(A \cap B \cap C) = P(A) \times P(B|A) \times P(C|A \cap B)$.

(b) $P(B \cap C|A) = P(B|A)P(C|B \cap A)$

Problem 8. An article in the British Medical Journal ["Comparison of Treatment of Renal Calculi by Operative Surgery, Percutaneous Nephrolithotomy, and Extracorporeal Shock Wave Lithotripsy" (1986, Vol. 82, pp. 879892)] provided the following discussion of success rates in kidney stone removals. Open surgery (OS) had a success rate of 78% (273/350) while a newer method, percutaneous nephrolithotomy (PN), had a success rate of 83% (289/350). This newer method looked better, but the results changed when stone diameter was considered. For stones with diameters less than two centimeters, 93% (81/87) of cases of open surgery were successful compared with only 87% (234/270) of cases of PN. For stones greater than or equal to two centimeters, the success rates were 73% (192/263) and 69% (55/80) for open surgery and PN, respectively. Open surgery is better for both stone sizes, but less successful in total. In 1951, E. H. Simpson pointed out this apparent contradiction (known as Simpsons Paradox) but the hazard still persists today. Explain how open surgery can be better for both stone sizes but worse in total. [Montgomery and Runger, 2010, Q2-115]