

HW 2 — Due: July 5

Lecturer: Prapun Suksompong, Ph.D.

Instructions

- (a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.
- (b) It is important that you try to solve all problems. (5 pt)
- (c) Late submission will be heavily penalized.
- (d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. If A , B , and C are disjoint events with $P(A) = 0.2$, $P(B) = 0.3$ and $P(C) = 0.4$, determine the following probabilities:

- (a) $P(A \cup B \cup C)$
- (b) $P(A \cap B \cap C)$
- (c) $P(A \cap B)$
- (d) $P((A \cup B) \cap C)$
- (e) $P(A^c \cap B^c \cap C^c)$

[Montgomery and Runger, 2010, Q2-75]

Problem 2. The sample space of a random experiment is $\{a, b, c, d, e\}$ with probabilities 0.1, 0.1, 0.2, 0.4, and 0.2, respectively. Let A denote the event $\{a, b, c\}$, and let B denote the event $\{c, d, e\}$. Determine the following:

- (a) $P(A)$
- (b) $P(B)$

(c) $P(A^c)$

(d) $P(A \cup B)$

(e) $P(A \cap B)$

$P(A \cap B) \in [0, 0] \cap [0, 1]$

i) $P(A \cap B) \in [0, 1]$

ii) $P(A \cap B) \leq \min\{P(A), P(B)\}$

iii) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\geq P(A) + P(B) - 1$

$[0, 0]$

[Montgomery and Runger, 2010, Q2-55]



Problem 3.



$[\frac{1}{6}, \frac{1}{2}]$



(a) Suppose that $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$. Find the range of possible values for $P(A \cap B)$.
Hint: Smaller than the interval $[0, 1]$. [Capinski and Zastawniak, 2003, Q4.21]

(b) Suppose that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. Find the range of possible values for $P(A \cup B)$.
Hint: Smaller than the interval $[0, 1]$. [Capinski and Zastawniak, 2003, Q4.22]

Problem 4. Let A and B be events for which $P(A)$, $P(B)$, and $P(A \cup B)$ are known. Express the following probabilities in terms of the three known probabilities above.

(a) $P(A \cap B)$

(b) $P(A \cap B^c) = P(A \setminus B) =$

(c) $P(B \cup (A \cap B^c))$

(d) $P(A^c \cap B^c)$

① $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

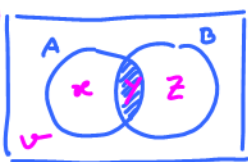
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

③ $P(A \cap B) = P(A) - P(A \setminus B)$

$P(A \cup B) - P(B)$

$= P(A) + P(B) - P(A \cup B)$

②



want to find y .

$x + y = P(A)$

$y + z = P(B)$

$x + y + z = P(A \cup B)$

$\rightarrow y?$