ECS 315: Probability and Random Processes **2013/1**

HW 12 — Due: Oct 4

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Problem 1 (Yates & Goodman, 2005, Q4.4.2)**.** Random variables X and Y have joint pdf

$$f_{X,Y}(x,y) = \begin{cases} cxy^2, & 0 \le x \le 1,\ 0 \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the constant c.

(b) Find $P[X > Y]$.

(c) Find $P[Y < X^2]$.

Hint: To find c, recall that joint pdf must integrate to 1. To find any probability specified by a condition involving two random variables, you need to integrate the joint pdf over the corresponding region (which is the region that contains all the points satisfying the condition).

Problem 2 (Yates & Goodman, 2005, Q4.5.1)**.** [Joint pdf to marginal pdf + Expectation] Random variables X and Y have the joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 1/2, & -1 \le x \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Sketch the region of nonzero probability. (The region of nonzero probability is defined to be the region where the joint pdf is positive.)

(b) What is $P[X > 0]$?

(c) What is $f_X(x)$?

(d) What is $\mathbb{E}X$?

Problem 3 (Yates & Goodman, 2005, Q4.6.8)**.** Random variables X and Y have joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 \le y \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $W = Y/X$.

(a) Find $F_W(w)$.

(b) Find $f_W(w)$.

(c) Find $\mathbb{E}[W]$.

Problem 4 (Yates & Goodman, 2005, Q4.7.12)**.** Random variables X and Y have joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 1/2, & -1 \le x \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $\mathbb{E}[XY]$.

(b) Find $\mathbb{E}\left[e^{X+Y}\right]$.

Problem 5 (Yates & Goodman, 2005, Q4.7.8)**.** Random variables X and Y have joint pdf

$$f_{X,Y}(x,y) = \begin{cases} (x+y)/3, & 0 \le x \le 1,\ 0 \le y \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) What are $\mathbb{E}X$ and $\operatorname{Var} X$?

(b) What are $\mathbb{E}Y$ and $\operatorname{Var} Y$?

(c) What is $\operatorname{Cov}[X,Y]$?

(d) What is $\mathbb{E}[X+Y]$?

(e) What is $\operatorname{Var}[X+Y]$?

Problem 6 (Yates & Goodman, 2005, Q4.11.1)**.** [Independence] Random variables X and Y have joint pdf

$$f_{X,Y}(x,y) = ce^{-(x^2/8)-(y^2/18)}.$$

(a) What is the constant c?

(b) Are X and Y independent?

Extra Question

Here is an extra question for those who want extra practice.

Problem 7 (Yates & Goodman, 2005, Q4.4.3)**.** Random variables X and Y have joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 6e^{-(2x+3y)}, & x \ge 0,\ y \ge 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $P[X > Y]$.

(b) Find $P[X+Y \le 1]$.