ECS 315: Probability and Random Processes	2013/1

HW 10 — Due: Sep 20

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. In wireless communications systems, fading is sometimes modeled by *lognormal* random variables. We say that a positive random variable Y is lognormal if $\ln Y$ is a normal random variable (say, with expected value m and variance σ^2). Find the pdf of Y.

Hint: First, recall that the ln is the natural log function (log base e). Let $X = \ln Y$. Then we know that $X \sim \mathcal{N}(m, \sigma^2)$. Can you write Y as a function of X? In class, we talked about a formula that turns pdf of X to pdf of Y. Use it here.

Problem 2. The input X and output Y of a system subject to random perturbations are described probabilistically by the following joint pmf matrix:

X \ Y	2	4	5
1	0.02	0.10	0.08
3	0.08	0.32	0.40

Use `MATLAB` or `Excel` to evaluate the following quantities:

(a) The marginal pmf $p_X(x)$

(b) The marginal pmf $p_Y(y)$

(c) $\mathbb{E}X$

(d) $\text{Var}\,X$

(e) $\mathbb{E}Y$

(f) $\text{Var}\,Y$

(g) $P[XY < 6]$

(h) $P[X = Y]$

Problem 3. The input X and output Y of a system subject to random perturbations are described probabilistically by the joint pmf $p_{X,Y}(x,y)$, where $x = 1, 2, 3$ and $y = 1, 2, 3, 4, 5$. Let P denote the joint pmf matrix whose i,j entry is $p_{X,Y}(i,j)$, and suppose that

$$P = \frac{1}{71} \begin{bmatrix} 7 & 2 & 8 & 5 & 4 \\ 4 & 2 & 5 & 5 & 9 \\ 2 & 4 & 8 & 5 & 1 \end{bmatrix}$$

(a) Find the marginal pmfs $p_X(x)$ and $p_Y(y)$.

(b) Find $\mathbb{E}X$

(c) Find $\mathbb{E}Y$

(d) Find $\text{Var}\,X$

(e) Find $\text{Var}\,Y$

Problem 4. Suppose $X \sim \text{binomial}(5, 1/3)$, $Y \sim \text{binomial}(7, 4/5)$, and $X \perp\!\!\!\perp Y$.

(a) A vector describing the pmf of X can be created by the MATLAB expression:

$$x = 0:5; \ pX = \text{binopdf}(x,5,1/3).$$

What is the expression that would give pX, a corresponding vector describing the pmf of Y?

(b) Use pX and pY from part (a), how can you create the joint pmf matrix in MATLAB? Do not use "for-loop", "while-loop", "if statement". Hint: Multiply them in an appropriate orientation.

(c) Use `MATLAB` to evaluate the following quantities. Again, do not use "for-loop", "while-loop", "if statement".

    (i) $\mathbb{E}X$

   (ii) $P[X = Y]$

  (iii) $P[XY < 6]$