

# ECS 315: Quiz 1 Solution

2013

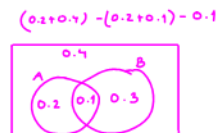
## Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group. Late submission will not be accepted.
3. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Name	ID
Prapun	555

1. Suppose you know that

$$P(A^c) = 0.7, \quad P(B^c) = 0.6, \quad P(A \cap B) = 0.1.$$

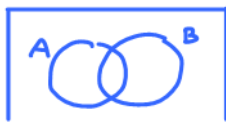


- a. Find  $P(A)$ .

Finite additivity

From  $A \cup A^c = \Omega$ , we have  $P(A) + P(A^c) = P(\Omega)$   
↑ disjoint union || ← Axiom P1  
1  
 $\Rightarrow P(A) = 1 - P(A^c) = 1 - 0.7 = 0.3.$

- b. Find  $P(A \cap B^c)$



From  $A = (A \cap B^c) \cup (A \cap B)$ , we have  $P(A) = P(A \cap B^c) + P(A \cap B)$   
↑ disjoint union  $P(A \cap B^c) = P(A) - P(A \cap B)$   
 $= 0.3 - 0.1 = 0.2.$

- c. Find  $P(A \cup B)$

Use the same reasoning as in (a) to get  $P(B) = 1 - P(B^c) = 1 - 0.6 = 0.4$   
 In class, we showed that  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .  
 So,  $P(A \cup B) = 0.3 + 0.4 - 0.1 = 0.6.$

2. When  $P(A^c) = 0.6$  and  $P(B^c) = 0.5$ , can  $P(A \cap B) = 0.55$ ? Please provide your reason.

$\Downarrow$   $\Downarrow P(B) = 1 - P(B^c) = 0.5$   
 $P(A) = 1 - P(A^c) = 0.4$

In class, we showed that  $P(A \cap B) \leq \min \{ P(A), P(B) \}$

Here,  $\min \{ P(A), P(B) \} = 0.4.$

However,  $P(A \cap B) = 0.55 > 0.4.$

Therefore,  $P(A \cap B) = 0.55$  is not possible.

$\uparrow$  This is actually easy to see.  
 $A \cap B \subset A$  and  $A \cap B \subset B$ .  
 Therefore,  
 $P(A \cap B) \leq P(A)$  and  $P(A \cap B) \leq P(B).$