4 Permutation of r types of n objects

Tuesday, July 10, 2012
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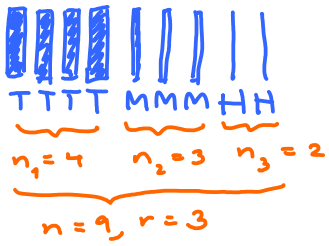
Goal: Try to understand the $\frac{n!}{n_1!n_2!\cdots n_r!}$

Consider counting all the permutation of



Step 1

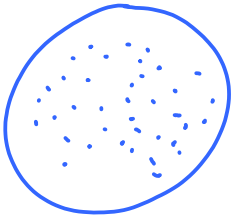
First, we will make <u>all</u> objects distinct.

So, instead of considering TTTT MMM HH,
we consider $T_1T_2T_3T_4M_1M_2M_3H_1H_2$.

It is then easy to see that we have

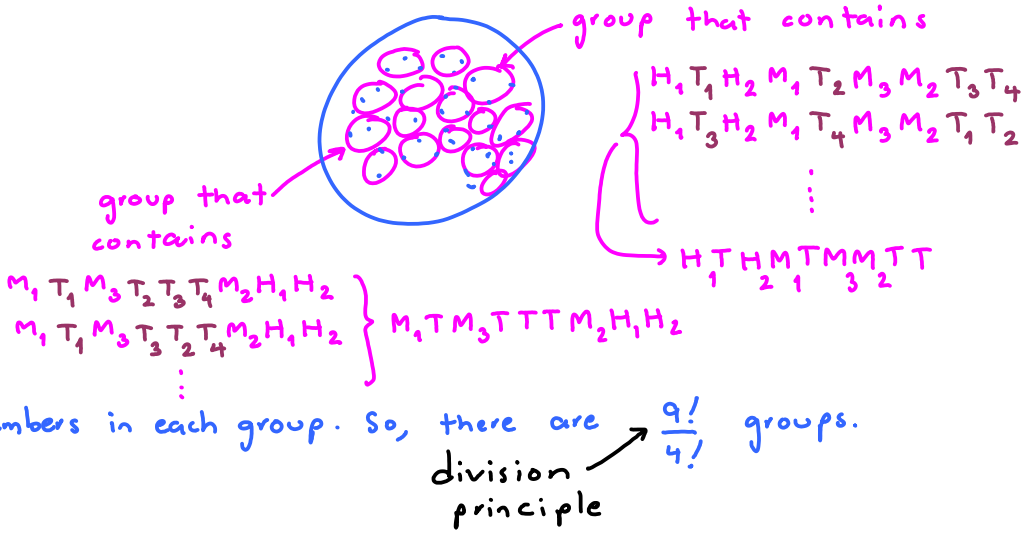
$9!$ possibilities

for the permutation.



Step 2

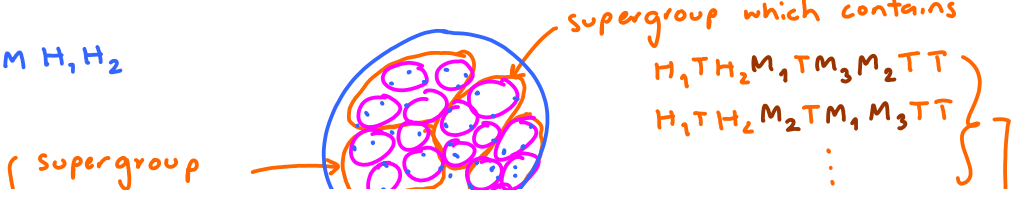
Now consider permutation of $TTTTM_1M_2M_3H_1H_2$.
Because the T's are now all the same, the permutation among the T's themselves do not create new permutation. So, we will group the possibilities that we got from step 1 according to the <u>locations</u> of the T's.

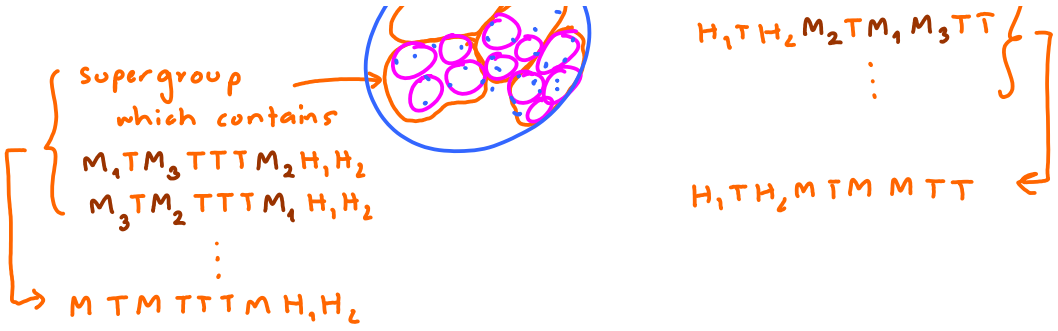


Step 3

Now consider permutation of

TTTT MMM H_1H_2





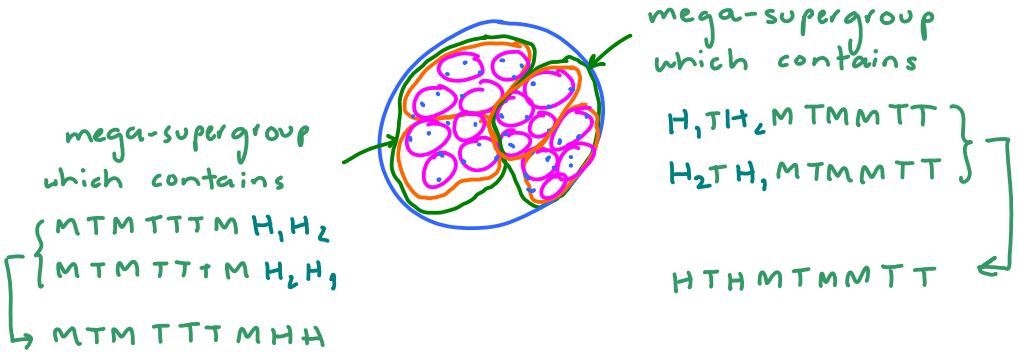
There are $3!$ groups in each supergroup.

So, there are $\frac{9!/4!}{3!} = \frac{9!}{4!3!}$ supergroups.

division principle

Step 4

Finally consider permutation of TTTTMMMHH



There are $2!$ supergroups in each mega-supergroup.

So, there are $\frac{9!/(4!3!)}{2!} = \frac{9!}{4!3!2!}$ mega-supergroups.

division principle.