ECS 315: Quiz 1 Solution

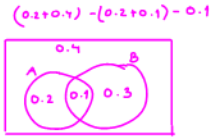
Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group. Late submission will not be accepted.
3. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
4. **Do not panic.**

Name	ID
Prapun	555

1. Suppose you know that

$$P(A^c)=0.7, \quad P(B^c)=0.6, \quad P(A\cap B)=0.1.$$



a. Find $P(A)$.

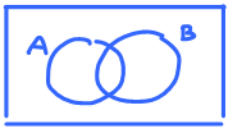
Finite additivity

From $A \cup A^c = \Omega$, we have $P(A)+P(A^c) = P(\Omega)$

↑ disjoint union

$\Rightarrow P(A) = 1 - P(A^c) = 1 - 0.7 = 0.3.$

b. Find $P(A\cap B^c)$



From $A = (A\cap B^c)\cup(A\cap B)$, we have $P(A) = P(A\cap B^c) + P(A\cap B)$

↑ disjoint union

$P(A\cap B^c) = P(A) - P(A\cap B)$

$= 0.3 - 0.1 = 0.2.$

c. Find $P(A\cup B)$

Use the same reasoning as in (a) to get $P(B) = 1 - P(B^c) = 1 - 0.6 = 0.4$

In class, we showed that $P(A\cup B) = P(A) + P(B) - P(A\cap B)$.

So, $P(A\cup B) = 0.3 + 0.4 - 0.1 = 0.6.$

2. When $P(A^c)=0.6$ and $P(B^c)=0.5$, can $P(A\cap B)=0.55$? Please provide your reason.

$P(A) = 1 - P(A^c) = 0.4$

$P(B) = 1 - P(B^c) = 0.5$

In class, we showed that $P(A\cap B) \le \min\{P(A), P(B)\}$

↑ This is actually easy to see.

$A\cap B \subset A$ and $A\cap B \subset B$.

Therefore,

$P(A\cap B) \le P(A)$ and $P(A\cap B) \le P(B)$.

Here, $\min\{P(A), P(B)\} = 0.4$.

However, $P(A\cap B) = 0.55 > 0.4$.

Therefore, $P(A\cap B) = 0.55$ is not possible.

Quiz 2

Friday, July 19, 2013 11:39 AM

Suppose a RV X has pmf

$$p_X(x) = \begin{cases} \frac{c}{x^2}, & x = -2, -1, 1, 2, 3 \\ 0, & \text{otherwise.} \end{cases}$$

3 pt (a) Find c

2 pt (b) Sketch the pmf

1 pt (c) $P[X = 2]$

2 pt (d) $P[X \leq 1.5]$

1 pt (e) $P[X > 2.5]$

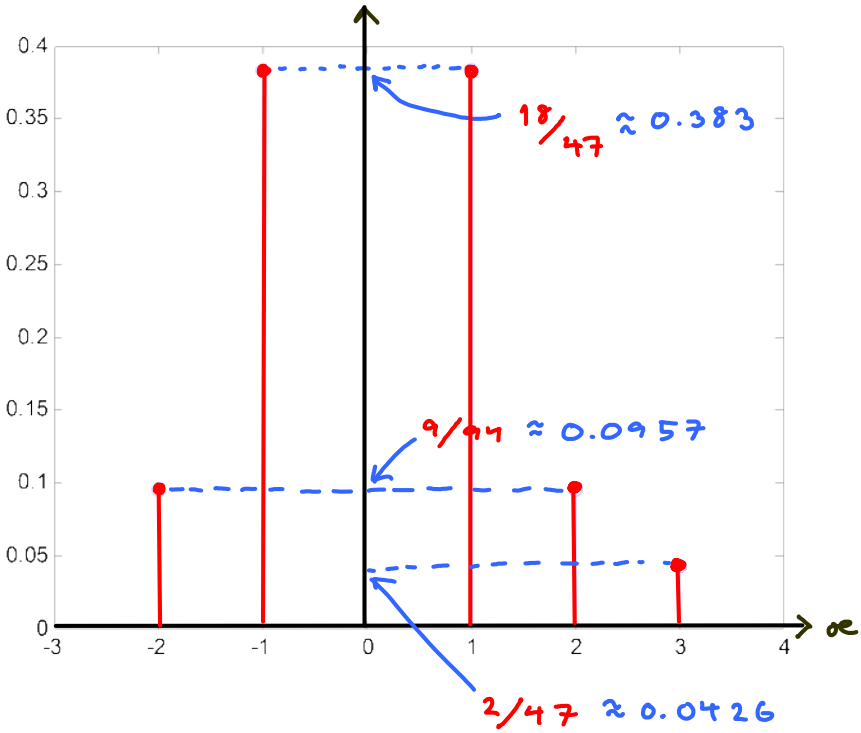
1 pt (f) Plot $P[X \leq x]$

Solutions:

(a) We use the fact that $\sum_x p_X(x) = 1$.

So, $\frac{c}{(-2)^2} + \frac{c}{(-1)^2} + \frac{c}{1^2} + \frac{c}{2^2} + \frac{c}{3^2} = 1 \Rightarrow c = \frac{18}{47} \approx 0.383$

(b) We simply plug-in $c = \frac{18}{47}$ into the given expression for pmf.



(c) $P[X = 2] = p_X(2) = \frac{9}{94} \approx 0.0957$

(d) $P[X \le 1.5] = P[X=-2] + P[X=-1] + P[X=1] = \frac{9}{94} + \frac{18}{47} + \frac{18}{47} = \frac{81}{94} \approx 0.8617$

(e) $P[X > 2.5] = P[X=3] = \frac{2}{47} \approx 0.0426$

(f) $P[X \le x]$ ← this is the CDF of X.

We add $p_X(\cdot)$ upto $p_X(a)$.