Tutorial on Sep 20, 2013

Friday, September 20, 2013
9:16 AM

Two RVs : joint pmf

① $p_{X,Y}(x,y) = \begin{cases} cxy, & x \in \{1,2\}, y \in \{1,3\}, \\ 0, & \text{otherwise.} \end{cases}$

② $P_{X,Y}$

x \ y	1	3
1	c	3c
2	2c	6c

=

x \ y	1	3
1	1/12	3/12
2	2/12	6/12

=

x \ y	1	3	
1	1/12	1/4	→ 1/3
2	1/6	1/2	→ 2/3
	↓ 1/4	↓ 3/4	

$p_{X,Y}(2,1) \overset{?}{=} p_X(2) \times p_Y(1)$

$\sum_x \sum_y p_{X,Y}(x,y) = c + 3c + 2c + 6c = 12c = 1$

$c = \frac{1}{12}$

marginal pmf's

$p_{X,Y}(x,y)$, $P_{X,Y}$ →

$p_X(x) = \begin{cases} 1/3, & x = 1 \\ 2/3, & x = 2 \\ 0, & \text{otherwise} \end{cases}$

$p_Y(y) = \begin{cases} 1/4, & y = 1 \\ 3/4, & y = 3 \\ 0, & \text{otherwise.} \end{cases}$

$\mathbb{E}X = 1 \times \frac{1}{3} + 2 \times \frac{2}{3} = \frac{5}{3}$

$\mathbb{E}[X^2] = 1 \times \frac{1}{3} + 4 \times \frac{2}{3} = \frac{9}{3} = 3$

$\mathrm{Var}\,X = \mathbb{E}[X^2] - (\mathbb{E}X)^2 = 3 - \frac{25}{9}$

$= \frac{2}{9}$

$\sigma_X = \frac{\sqrt{2}}{3}$

$= \mathbb{E}\left[(X - \mathbb{E}X)^2\right] = \frac{1}{3}\left(-\frac{2}{3}\right)^2 + \frac{2}{3}\left(\frac{1}{3}\right)^2 = \frac{6}{27} = \frac{2}{9}$

$\sigma_X = \sqrt{\mathbb{E}\left[(X - \mathbb{E}X)^2\right]}$

X and Y are not identically distributed.

X and Y are independent

X and Y are not i.i.d.

$P[XY > 2]$

$x \times y$

x \ y	1	3
1	1	3
2	2	6

x		y	
1	1	1	3
2	2	1	3