Probability and Random Processes
ECS 315

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Discrete Random Variable

Office Hours:
Rangsit Library:
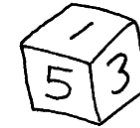   **Tuesday 16:20-17:20**
BKD3601-7:
    Thursday 16:00-17:00

Roll a fair dice. Record the result.

X ~ Uniform({1,2,...,6})

```
>> X = randi(6)

X =

      5
```

Again, roll a fair dice. Record the result.
```
>> X = randi(6)

X =

      6
```

Again, roll a fair dice. Record the result.
```
>> X = randi(6)

X =

      1
```

Again, roll a fair dice. Record the result.
```
>> X = randi(6)

X =

      6
```

Again, roll a fair dice. Record the result.
```
>> X = randi(6)

X =

      4
```

Again, roll a fair dice. Record the result.
```
>> X = randi(6)

X =

      1
```

```
>> X = randi(6,20,10)

X =
```

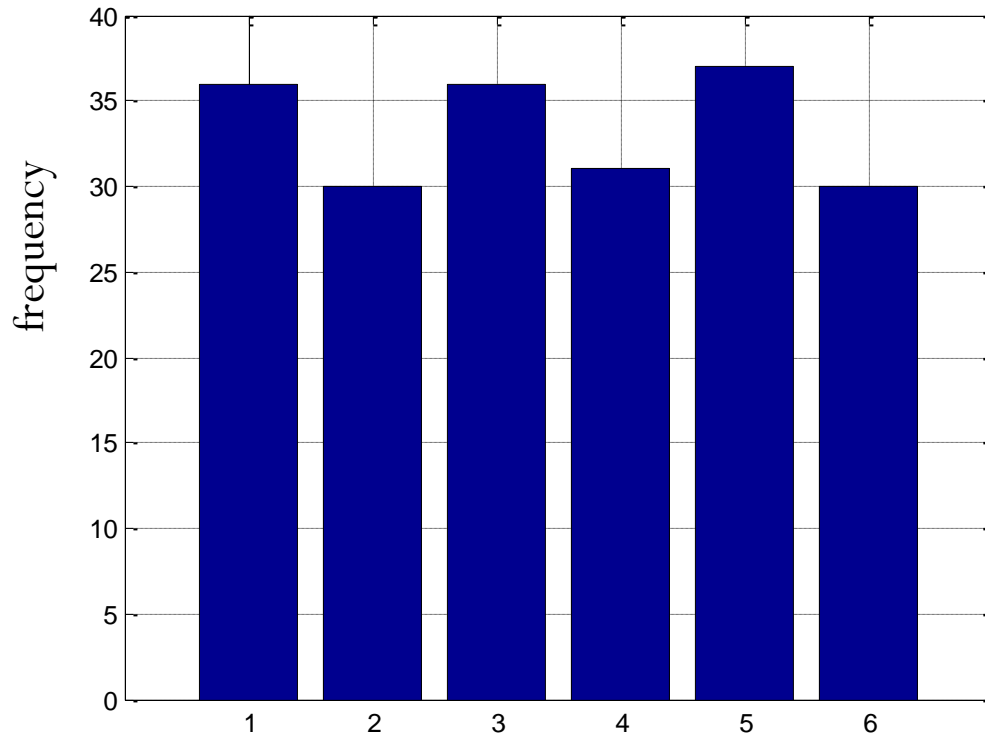Generate X 200 times. Put the results in a table of size 20×10

2	5	3	4	5	2	1	2	6	3
4	3	4	1	5	4	3	1	6	4
6	4	5	1	3	5	2	2	4	5
6	2	5	2	4	5	5	2	1	1
1	5	2	6	1	3	3	3	2	6
6	1	5	2	1	1	6	1	3	5
6	2	4	5	4	2	2	6	5	3
3	1	1	2	5	6	2	6	1	3
5	1	1	6	6	1	1	3	1	3
1	5	3	3	1	5	1	3	2	2
3	5	6	2	4	4	6	3	4	4
6	2	3	2	3	6	4	6	5	4
5	6	4	4	1	1	4	3	4	5
6	1	2	3	3	3	1	1	3	5
4	3	5	3	1	1	6	5	4	4
1	3	2	5	5	6	4	3	2	3
6	5	4	4	2	1	3	2	5	5
6	5	5	4	4	5	4	3	2	4
5	2	6	6	1	5	3	1	5	3
5	3	6	2	4	6	1	1	2	6

Histogram

$[N, x] = \text{hist}(\text{reshape}(X, 1, \text{prod}(\text{size}(X))), 1:6)$
$\text{bar}(x, N)$
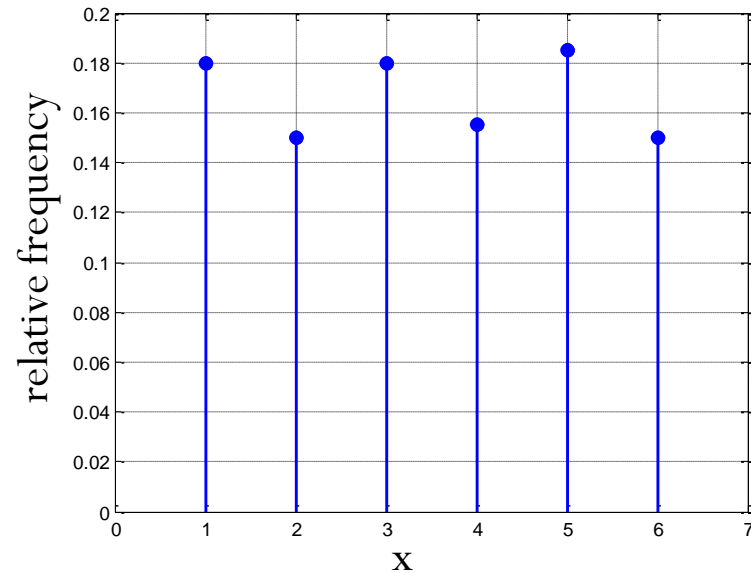Grid on

Relative Frequency
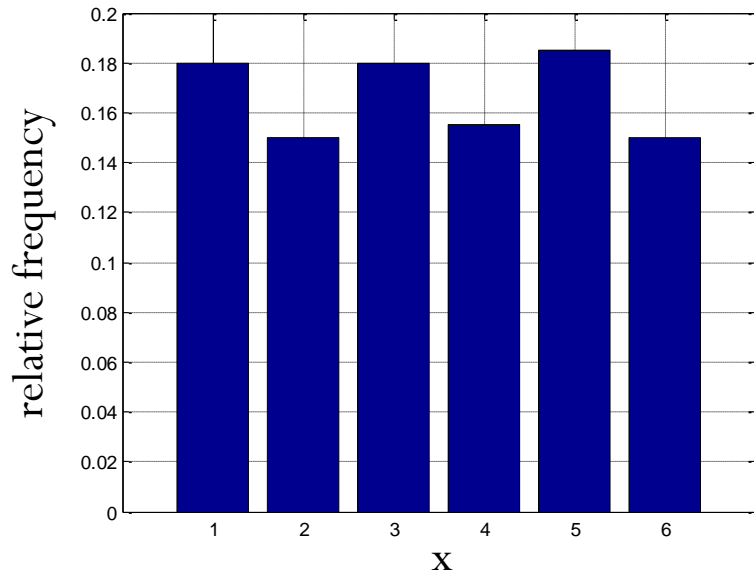
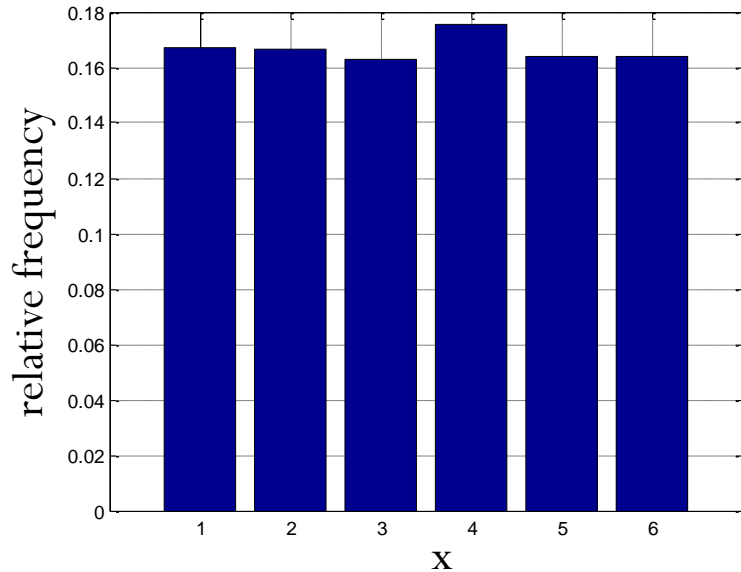$rf = N/prod(size(X))$
bar(x,rf)
grid on
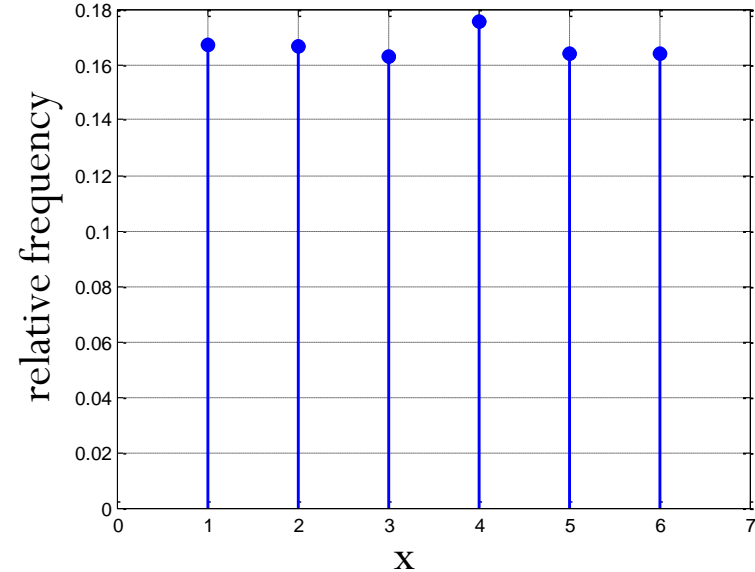
stem(x,rf,'filled','LineWidth',1.5)
grid on

With larger number of samples

rf = N/prod(size(X))
bar(x,rf)
grid on

stem(x,rf,'filled','LineWidth',1.5)
grid on

X = randi(6,100,100);

Flip an unfair coin 10 times. (The probability of getting heads for each time is 0.3.)
Count the number of heads.

$$X \sim \text{binomial}(\underset{n}{10}, \underset{p}{0.3})$$

```
>> X = binornd(10,0.3)

X =

     3
```
Again, flip an unfair coin 10 times. Count #H.
```
>> X = binornd(10,0.3)

X =

     2
```
Again, flip an unfair coin 10 times. Count #H.
```
>> X = binornd(10,0.3)

X =

     2
```
Again, flip an unfair coin 10 times. Count #H.
```
>> X = binornd(10,0.3)

X =

     5
```
Again, flip an unfair coin 10 times. Count #H.
```
>> X = binornd(10,0.3)

X =

     1
```
Again, flip an unfair coin 10 times. Count #H.
```
>> X = binornd(10,0.3)

X =

     4
```
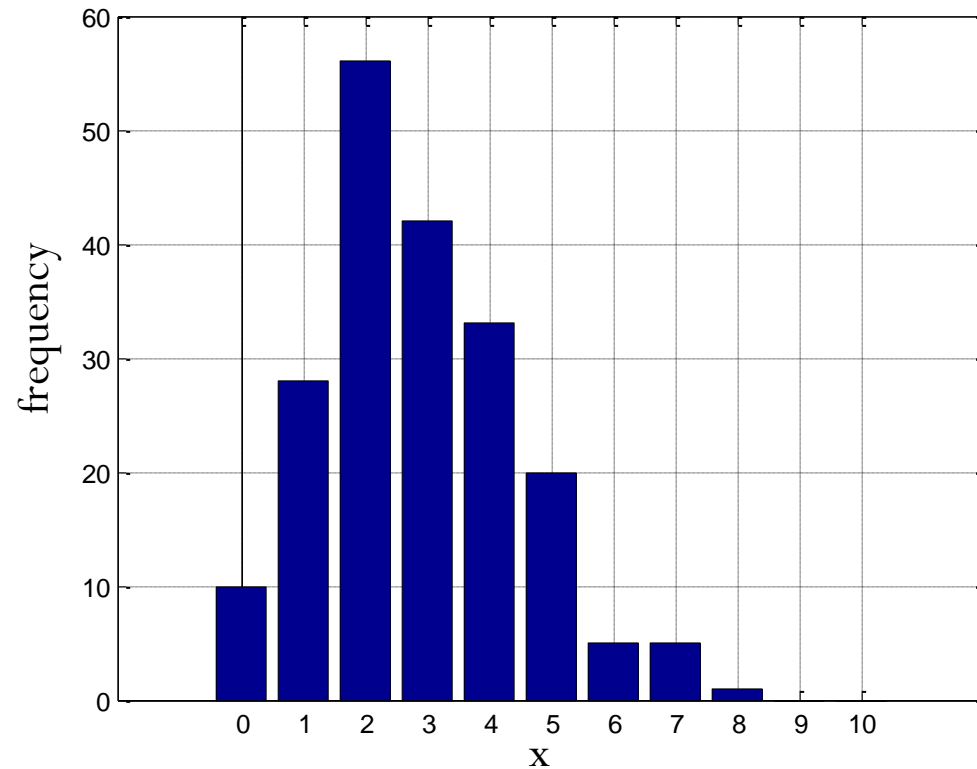
```
>> X = binornd(10,0.3,20,10)

X =
```

Generate X 200 times. Put the results in a table of size 20×10

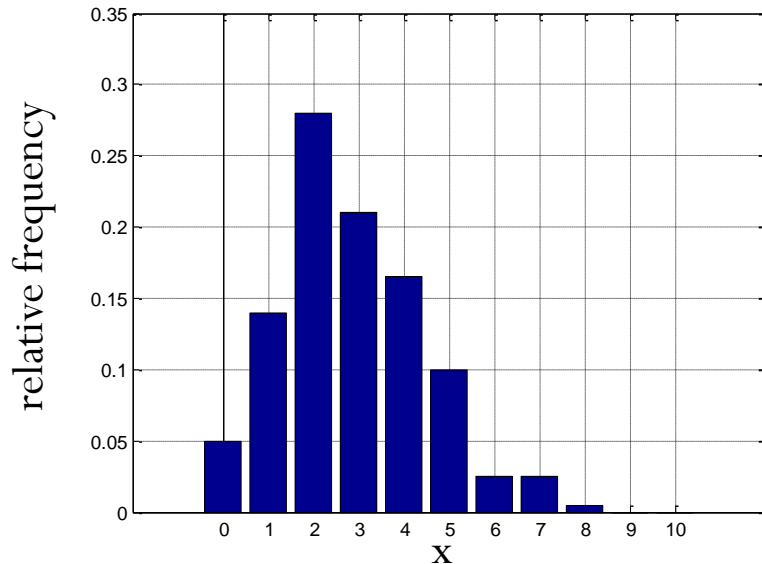|									
3	4	4	5	7	2	2	2	2	1
3	5	3	1	0	4	2	1	2	3
5	2	2	6	4	2	2	4	3	1
1	2	2	4	2	4	3	3	3	5
4	1	4	3	3	4	2	2	2	2
2	1	3	1	5	2	5	2	1	2
4	0	3	3	2	1	2	1	3	1
4	4	0	2	3	6	2	3	1	1
5	0	3	3	7	1	3	1	3	8
1	2	4	4	1	5	2	4	5	1
5	2	4	6	3	2	3	3	5	0
2	4	0	0	2	2	3	2	0	2
4	3	3	2	2	2	1	2	7	4
2	4	2	1	3	3	4	3	5	2
5	3	2	3	4	2	3	3	1	2
2	6	2	3	4	4	4	5	6	7
5	1	2	4	3	3	0	5	0	2
1	4	1	3	1	4	2	4	2	4
5	2	2	3	3	5	3	5	2	1
4	2	4	3	2	5	7	2	3	1

Histogram: X ~ binomial(10,0.3)

[N, x] = hist(reshape(X,1,prod(size(X))),0:10)
bar(x,N)
Grid on

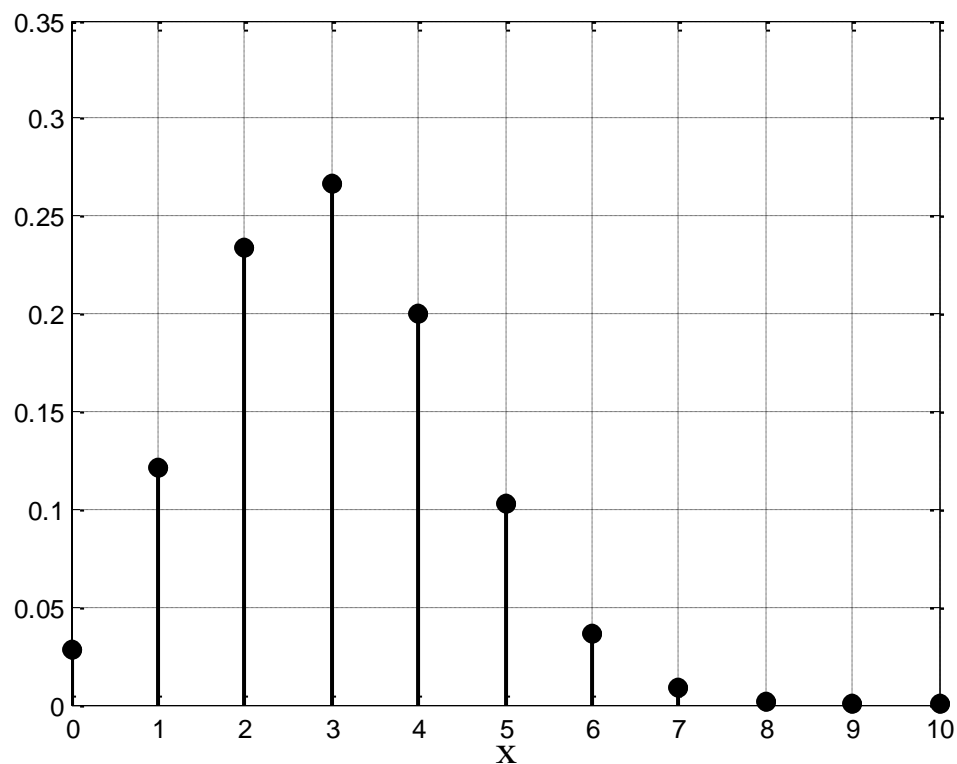# Relative Freq.: X ~ binomial(10,0.3)

$rf = N/prod(size(X))$
bar(x,rf)
grid on
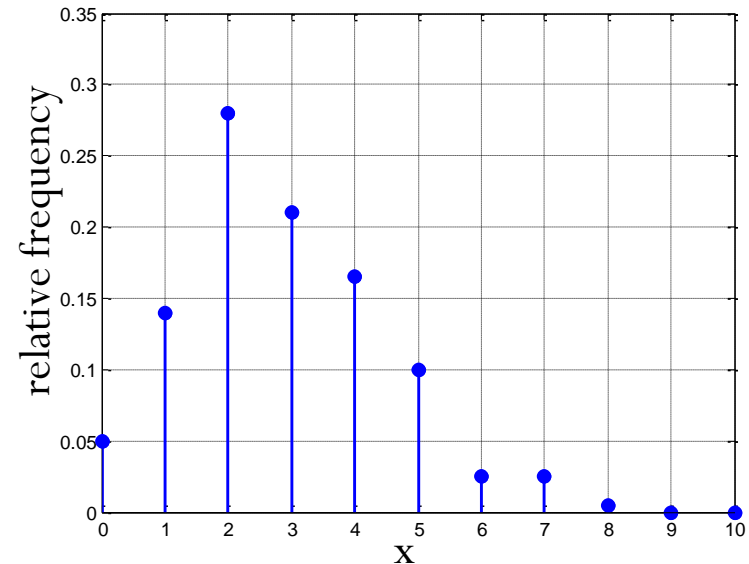
stem(x,rf,'filled','LineWidth',1.5)
grid on

pmf for X ~ binomial(10,0.3)

$$p_X(x) = \binom{10}{x} 0.3^x (1 - 0.3)^{10-x}$$

p = binopdf(x,10,0.3)
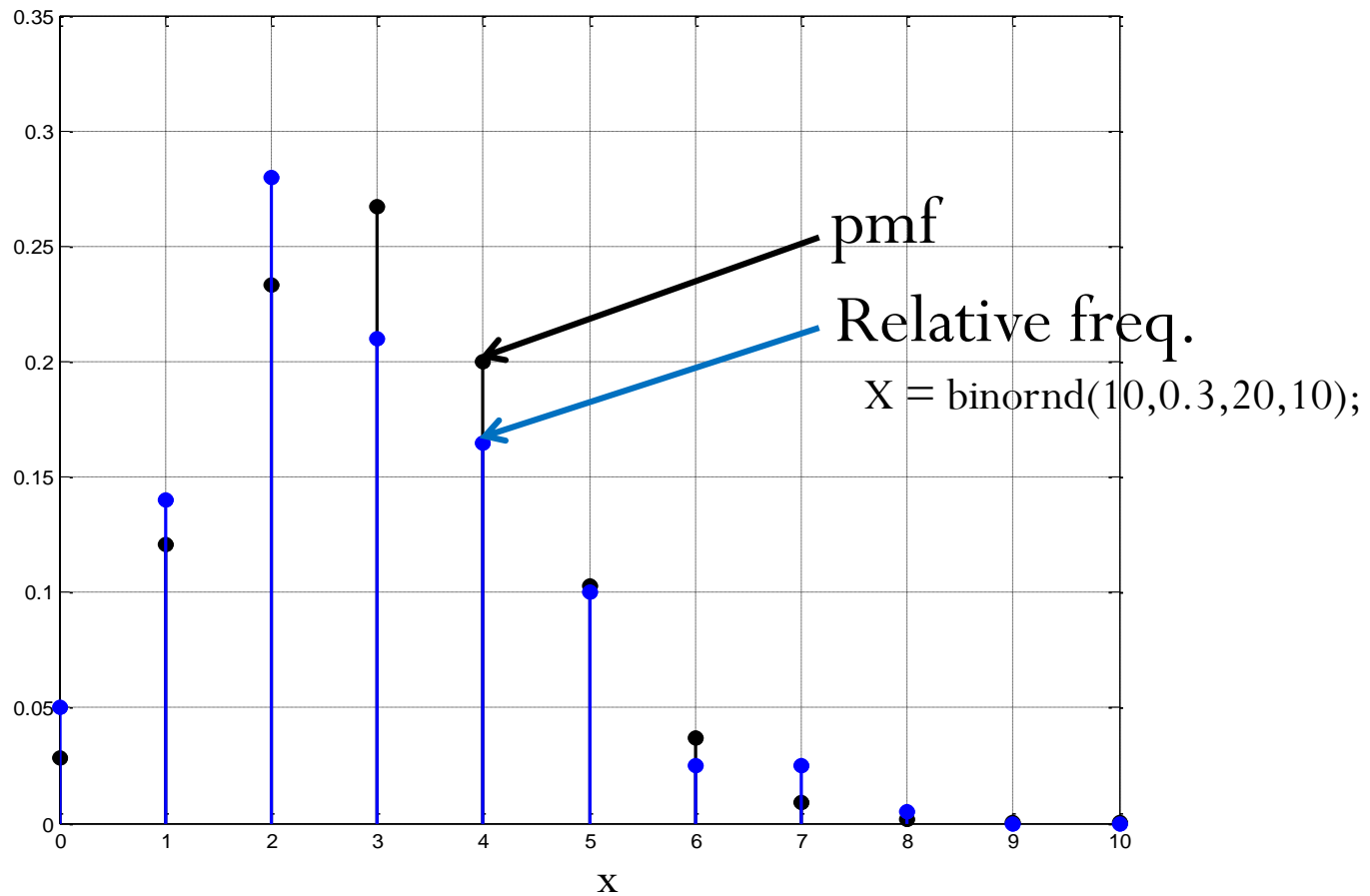
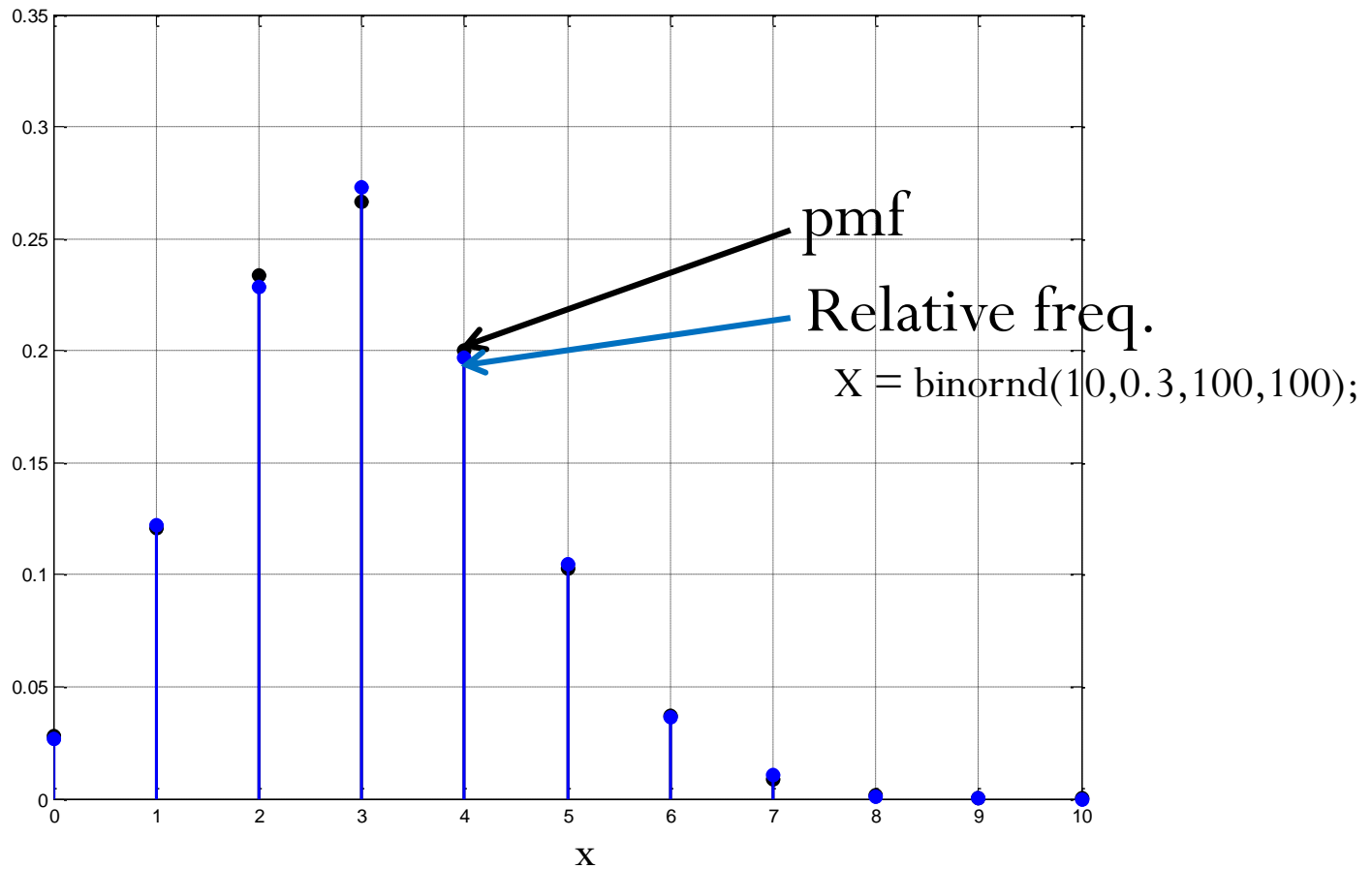stem(x,p,'k','filled','LineWidth',1.5); grid on

X ~ binomial(10,0.3)

pmf

Relative freq.

$X = \text{binornd}(10,0.3,20,10);$

x

X ~ binomial(10,0.3)

pmf

Relative freq.

$X = binornd(10, 0.3, 100, 100);$

Publishing Success

- **Publishing success** is so **unpredictable** that even if our novel is destined for the best-seller list, numerous publishers could miss the point and send those letters that say thanks but no thanks.

- In fact, many books destined for great success had to survive not just rejection, but repeated rejection.

- J. K. **Rowling**'s first **Harry Potter** manuscript was rejected by **nine** publishers.

- Lesson: Suppose four publishers have rejected your manuscript.
  - Your intuition and the bad feeling in the pit of your stomach might say that the rejections by all those publishing experts mean your manuscript is no good.
  - We all know from experience that if several tosses of a coin come up heads, it doesn't mean we are tossing a two-headed coin.

Box Office Success

- Hollywood's unpredictability
- Does luck play a far more important role in box office success (and failure) than people imagine?
- There are reasons for a film's box office performance
  - but those reasons are so complex and the path from green light to opening weekend so vulnerable to unforeseeable and uncontrollable influences that
  - educated guesses about an unmade film's potential aren't much better than flips of a coin.
- Studio executive David Picker:
  - "If I had said yes to all the projects I turned down, and no to all the other ones I took, it would have worked out about the same."

Don't give up

- Successful people in every field are almost universally members of a certain set—the set of people who don't give up.

Poisson Approximation

- Consider n Bernoulli trials.
- Assume success probability for each trial is $1/n$.

$$\#\text{successes} \sim \text{binomial}\left(n, \frac{1}{n}\right)$$

when n is large

$$\approx \text{Poisson}(1)$$

$$e^{-\alpha}\frac{\alpha^k}{k!} \overset{\alpha=1}{=} \frac{1}{k!e}$$

$P[\#\text{successes} \geq 1]$

$1 - \frac{1}{e} \approx 0.6321$

$P[\#\text{successes} = 1]$

$\frac{1}{e} \approx 0.3679$

$P[\#\text{successes} = 0]$

$P[\#\text{successes} = 2]$

$\frac{1}{2e} \approx 0.1839$

$P[\#\text{successes} = 3]$

$\frac{1}{3!e}$