

## HW 7 — Due: Oct 4

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**Instructions**

- (a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.
- (b) It is important that you try to solve all problems. (5 pt)
- (c) Late submission will be heavily penalized.
- (d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
- (e) For problems that use **MATLAB**, you must also print and submit your m-file along with the numerical results. In the beginning part of your code, there should be one commented line that indicates your name and student ID.

**Problem 1.** The input  $X$  and output  $Y$  of a system subject to random perturbations are described probabilistically by the following joint pmf matrix:

$x \backslash y$	2	4	5
1	[0.02	0.10	0.08]
3	[0.08	0.32	0.40]

- (a) Use **MATLAB** to evaluate the following quantities:
  - (i)  $\mathbb{E}[XY]$
  - (ii)  $\mathbb{E}[(X - 3)(Y - 2)]$
  - (iii)  $\mathbb{E}[X(Y^3 - 11Y^2 + 38Y)]$
  - (iv)  $\text{Cov}[X, Y]$
  - (v)  $\rho_{X,Y}$

- (b) Find  $\rho_{X,X}$
- (c) Calculate the following quantities using the values of  $\text{Var } X$ ,  $\text{Cov}[X, Y]$ , and  $\rho_{X,Y}$  that you got earlier.
- $\text{Cov}[3X + 4, 6Y - 7]$
  - $\rho_{3X+4, 6Y-7}$
  - $\text{Cov}[X, 6X - 7]$
  - $\rho_{X, 6X-7}$

**Problem 2.** Suppose  $\text{Var } X = 5$ . Find  $\text{Cov}[X, X]$  and  $\rho_{X,X}$ .

**Problem 3.** The input  $X$  and output  $Y$  of a system subject to random perturbations are described probabilistically by the joint pmf  $p_{X,Y}(x, y)$ , where  $x = 1, 2, 3$  and  $y = 1, 2, 3, 4, 5$ . Let  $P$  denote the joint pmf matrix whose  $i, j$  entry is  $p_{X,Y}(i, j)$ , and suppose that

$$P = \frac{1}{71} \begin{bmatrix} 7 & 2 & 8 & 5 & 4 \\ 4 & 2 & 5 & 5 & 9 \\ 2 & 4 & 8 & 5 & 1 \end{bmatrix}$$

- Find  $\mathbb{E}[XY]$
- Find  $\mathbb{E}[(3X + 4)(6Y - 7)]$
- Find  $\text{Cov}[X, Y]$
- Find  $\text{Cov}[3X + 4, 6Y - 7]$

**Problem 4.** Suppose we know that  $\sigma_X = \frac{\sqrt{21}}{10}$ ,  $\sigma_Y = \frac{4\sqrt{6}}{5}$ ,  $\rho_{X,Y} = -\frac{1}{\sqrt{126}}$ .

- Find  $\text{Var}[X + Y]$ .
- Find  $\mathbb{E}[(Y - 3X + 5)^2]$ . Assume  $\mathbb{E}[Y - 3X + 5] = 1$ .

**Problem 5.** Suppose  $X \sim \text{binomial}(5, 1/3)$ ,  $Y \sim \text{binomial}(7, 4/5)$ , and  $X \perp\!\!\!\perp Y$ . Use MATLAB to evaluate the following quantities.

- $\mathbb{E}[(X - 3)(Y - 2)]$
- $\mathbb{E}[X(Y^3 - 11Y^2 + 38Y)]$
- $\text{Cov}[X, Y]$
- $\rho_{X,Y}$