

HW 3 — Due: Aug 10

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Instructions

- (a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.
- (b) It is important that you try to solve all problems. (5 pt)
- (c) Late submission will be heavily penalized.
- (d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Suppose that for the general population, 1 in 5000 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive (+) or negative (-) response. Suppose the test gives the correct answer 99% of the time.

- (a) What is $P(-|H)$, the conditional probability that a person tests negative given that the person does have the HIV virus?
- (b) What is $P(H|+)$, the conditional probability that a randomly chosen person has the HIV virus given that the person tests positive?

Problem 2. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Let A denote the event that the design color is red and let B denote the event that the font size is not the smallest one.

- (a) Use classical probability to evaluate $P(A)$, $P(B)$ and $P(A \cap B)$. Show that the two events A and B are independent by checking whether $P(A \cap B) = P(A)P(B)$.
- (b) Using the values of $P(A)$ and $P(B)$ from the previous part and the fact that $A \perp\!\!\!\perp B$, calculate the following probabilities.
 - (i) $P(A \cup B)$
 - (ii) $P(A \cup B^c)$

(iii) $P(A^c \cup B^c)$

[Montgomery and Runger, 2010, Q2-84]

Problem 3. Anne and Betty go fishing. Find the conditional probability that Anne catches no fish given that at least one of them catches no fish. Assume they catch fish independently and that each has probability $0 < p < 1$ of catching no fish. [Gubner, 2006, Q2.62]

Problem 4. Show that if A and B are independent events, then so are A and B^c , A^c and B , and A^c and B^c .

Problem 5. In this question, each experiment has equiprobable outcomes.

(a) Let $\Omega = \{1, 2, 3, 4\}$, $A_1 = \{1, 2\}$, $A_2 = \{1, 3\}$, $A_3 = \{2, 3\}$.

(i) Determine whether $P(A_i \cap A_j) = P(A_i)P(A_j)$ for all $i \neq j$.

(ii) Check whether $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$.

(iii) Are A_1, A_2 , and A_3 independent?

(b) Let $\Omega = \{1, 2, 3, 4, 5, 6\}$, $A_1 = \{1, 2, 3, 4\}$, $A_2 = A_3 = \{4, 5, 6\}$.

(i) Check whether $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$.

(ii) Check whether $P(A_i \cap A_j) = P(A_i)P(A_j)$ for all $i \neq j$.

(iii) Are A_1, A_2 , and A_3 independent?

Problem 6 (Majority Voting in Digital Communication). A certain binary communication system has a bit-error rate of 0.1; i.e., in transmitting a single bit, the probability of receiving the bit in error is 0.1. To transmit messages, a three-bit repetition code is used. In other words, to send the message 1, 111 is transmitted, and to send the message 0, 000 is transmitted. At the receiver, if two or more 1s are received, the decoder decides that message 1 was sent; otherwise, i.e., if two or more zeros are received, it decides that message 0 was sent.

Assuming bit errors occur independently, find the probability that the decoder puts out the wrong message.

[Gubner, 2006, Q2.62]

Problem 7. In an experiment, A , B , C , and D are events with probabilities $P(A \cup B) = \frac{5}{8}$, $P(A) = \frac{3}{8}$, $P(C \cap D) = \frac{1}{3}$, and $P(C) = \frac{1}{2}$. Furthermore, A and B are disjoint, while C and D are independent.

(a) Find

(i) $P(A \cap B)$

(ii) $P(B)$

(iii) $P(A \cap B^c)$

(iv) $P(A \cup B^c)$

(b) Are A and B independent?

(c) Find

(i) $P(D)$

(ii) $P(C \cap D^c)$

(iii) $P(C^c \cap D^c)$

(iv) $P(C|D)$

(v) $P(C \cup D)$

(vi) $P(C \cup D^c)$

(d) Are C and D^c independent?