

HW 2 — Due: July 31

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Instructions

- (a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.
- (b) It is important that you try to solve all problems. (5 pt)
- (c) Late submission will be heavily penalized.
- (d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. If A , B , and C are disjoint events with $P(A) = 0.2$, $P(B) = 0.3$ and $P(C) = 0.4$, determine the following probabilities:

- (a) $P(A \cup B \cup C)$
- (b) $P(A \cap B \cap C)$
- (c) $P(A \cap B)$
- (d) $P((A \cup B) \cap C)$
- (e) $P(A^c \cap B^c \cap C^c)$

[Montgomery and Runger, 2010, Q2-75]

Problem 2. The sample space of a random experiment is $\{a, b, c, d, e\}$ with probabilities 0.1, 0.1, 0.2, 0.4, and 0.2, respectively. Let A denote the event $\{a, b, c\}$, and let B denote the event $\{c, d, e\}$. Determine the following:

- (a) $P(A)$
- (b) $P(B)$

- (c) $P(A^c)$
- (d) $P(A \cup B)$
- (e) $P(A \cap B)$

[Montgomery and Runger, 2010, Q2-55]

Problem 3.

- (a) Suppose that $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$. Find the range of the possible value for $P(A \cap B)$. Hint: Smaller than the interval $[0, 1]$. [Capinski and Zastawniak, 2003, Q4.21]
- (b) Suppose that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. Find the range of the possible value for $P(A \cup B)$. Hint: Smaller than the interval $[0, 1]$. [Capinski and Zastawniak, 2003, Q4.22]

Problem 4. Let A and B be events for which $P(A)$, $P(B)$, and $P(A \cup B)$ are known. Express the following probabilities in terms of the three known probabilities above.

- (a) $P(A \cap B)$
- (b) $P(A \cap B^c)$
- (c) $P(B \cup (A \cap B^c))$
- (d) $P(A^c \cap B^c)$

Problem 5.

- (a) Suppose that $P(A|B) = 0.4$ and $P(B) = 0.5$. Determine the following:
 - (i) $P(A \cap B)$
 - (ii) $P(A^c \cap B)$

[Montgomery and Runger, 2010, Q2-105]

- (b) Suppose that $P(A|B) = 0.2$, $P(A|B^c) = 0.3$ and $P(B) = 0.8$. What is $P(A)$? [Montgomery and Runger, 2010, Q2-106]

Problem 6. Due to an Internet configuration error, packets sent from New York to Los Angeles are routed through El Paso, Texas with probability $3/4$. Given that a packet is routed through El Paso, suppose it has conditional probability $1/3$ of being dropped. Given that a packet is not routed through El Paso, suppose it has conditional probability $1/4$ of being dropped.

- (a) Find the probability that a packet is dropped.
- (b) Find the conditional probability that a packet is routed through El Paso given that it is not dropped.

[Gubner, 2006, Ex.1.20]

Problem 7. You have two coins, a fair one with probability of heads $\frac{1}{2}$ and an unfair one with probability of heads $\frac{1}{3}$, but otherwise identical. A coin is selected at random and tossed, falling heads up. How likely is it that it is the fair one? [Capinski and Zastawniak, 2003, Q7.28]

Problem 8. You have three coins in your pocket, two fair ones but the third biased with probability of heads p and tails $1-p$. One coin selected at random drops to the floor, landing heads up. How likely is it that it is one of the fair coins? [Capinski and Zastawniak, 2003, Q7.29]

Problem 9. Someone has rolled a fair dice twice. You know that one of the rolls turned up a face value of six. What is the probability that the other roll turned up a six as well? [Tijms, 2007, Example 8.1, p. 244]

Hint: Not $\frac{1}{6}$.

Problem 10.

- (a) Suppose that $P(A|B) = 1/3$ and $P(A|B^c) = 1/4$. Find the range of the possible values for $P(A)$.
- (b) Suppose that C_1, C_2 , and C_3 partition Ω . Furthermore, suppose we know that $P(A|C_1) = 1/3$, $P(A|C_2) = 1/4$ and $P(A|C_3) = 1/5$. Find the range of the possible values for $P(A)$.