HW1 Q7

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12:13 AM

Comments for Q7 on HW1

It should be noted that this is a problem on the "classical probability" part of the class. So, the technique presented in the solution is based on classical probability. In particular, we start by first defining a finite sample space whose outcomes are equally likely.

I have seen many submitted works that try to avoid using classical probability. There is nothing wrong with applying different techniques. However, it turns out that you will need to explain even more than you need to at this point in order to have a valid solution.

Many of you may solve the question by the following argument: (or its variants)

Let C_i be the result of the i^{th} button

A_i be the event that C_i = red

Because the button has two sides, it is reasonable to assume that "C_i = red" and "C_i = white" are equally likely.

Hence, $P(A_i) = P(A_i^c) = \frac{1}{2}$.

So far, there is no problem with the argument.

However, if you say

$$\underbrace{P(A_1 \cap A_2 \cap A_3)}_{\text{all red}} = \underbrace{\frac{1}{2}}_{P(A_1)} \times \underbrace{\frac{1}{2}}_{P(A_2)} \times \underbrace{\frac{1}{2}}_{P(A_3)} = \frac{1}{8},$$

you are skipping one step of the argument.

In general, if we can show or convince ourselves that three events A, B, and C are independent,

then $P(A \cap B \cap C) = P(A)P(B)P(C)$.

The concept of independence is studied in Section 6.2.

Without independence, we can't blindly write

$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$.

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)\,P(A_2)\,P(A_3).$$

Of course, in this question, the result of one buttons is independent of other buttons. So, it is OK to use . The problem is that you need to explicitly state that you can do this because you know that the three events are independent.

Similarly, independence should be explicitly stated before you write

$$\underbrace{P(A_1^c \cap A_2^c \cap A_3^c)}_{\text{all white}} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}.$$

Some of you calculate the final answer by

$$P\Big(\underbrace{(A_1 \cap A_2 \cap A_3) \cup (A_1^c \cap A_2^c \cap A_3^c)}_{\text{"all red" or "all white"}}\Big) = P(A_1 \cap A_2 \cap A_3) + P(A_1^c \cap A_2^c \cap A_3^c)$$

$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}.$$

To write $P(A \cup B) = P(A) + P(B)$, one will need to show that the events A and B are disjoint.
Now, $A_1 \cap A_2 \cap A_3$ and $A_1^c \cap A_2^c \cap A_3^c$ are always disjoint and therefore, it is OK to split the probability into the sum above. However, you need to explicitly say that you can safely do this because you know the two events are disjoint.

Alternatively, one can say that "all red" and "all white" are different outcomes of the experiment so, the corresponding events are automatically disjoint. However, this requires defining the sample space with 8 outcomes as we did in the solution.

Similarly, to write $|A \cup B| = |A| + |B|$, you also need to first show that A and B are disjoint.