

A result from calculus

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$$\lim_{x \rightarrow \infty} \left(1 + \frac{c}{x}\right)^x = e^c$$

Consider the function $\ln f(x)$ where $f(x) = \left(1 + \frac{c}{x}\right)^x$.

$$\text{Note that } \ln f(x) = x \ln \left(1 + \frac{c}{x}\right) = \frac{\ln \left(1 + \frac{c}{x}\right)}{1/x}$$

and

$$\lim_{x \rightarrow \infty} \ln f(x) = \lim_{x \rightarrow \infty} \frac{\left(1 + \frac{c}{x}\right)^{-1} c \left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}} = c$$

L'Hopital

$$\text{Therefore, } \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} e^{\ln f(x)} = e^{\lim_{x \rightarrow \infty} \ln f(x)} = e^c$$