Probability and Random Processes

ECS 315

Dr. Prapun Suksompong

prapun@siit.tu.ac.th

6.2 Independence

Office Hours:

BKD 3601-7

Monday	14:40-16:00
Friday	14:00-16:00

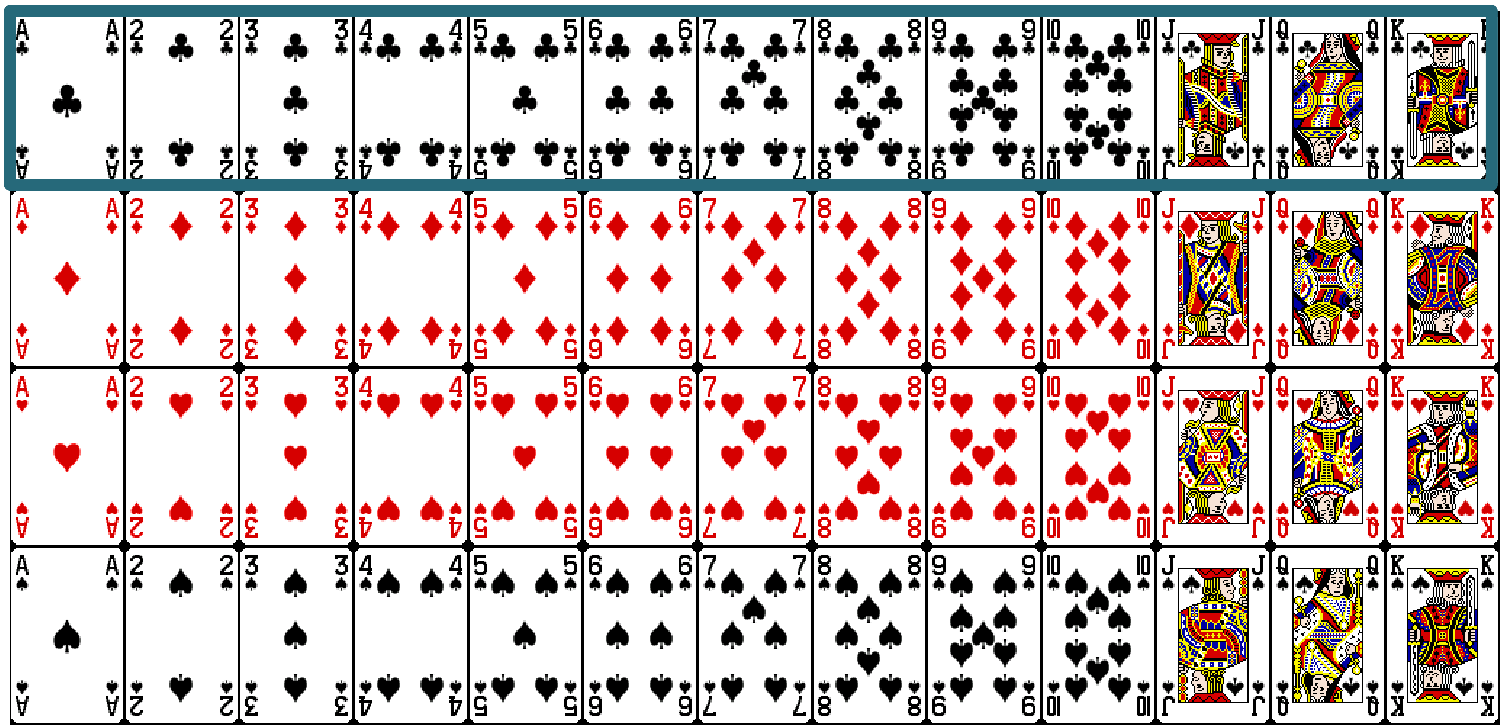
Example: Club & Black

clubs

diamonds

hearts

spades



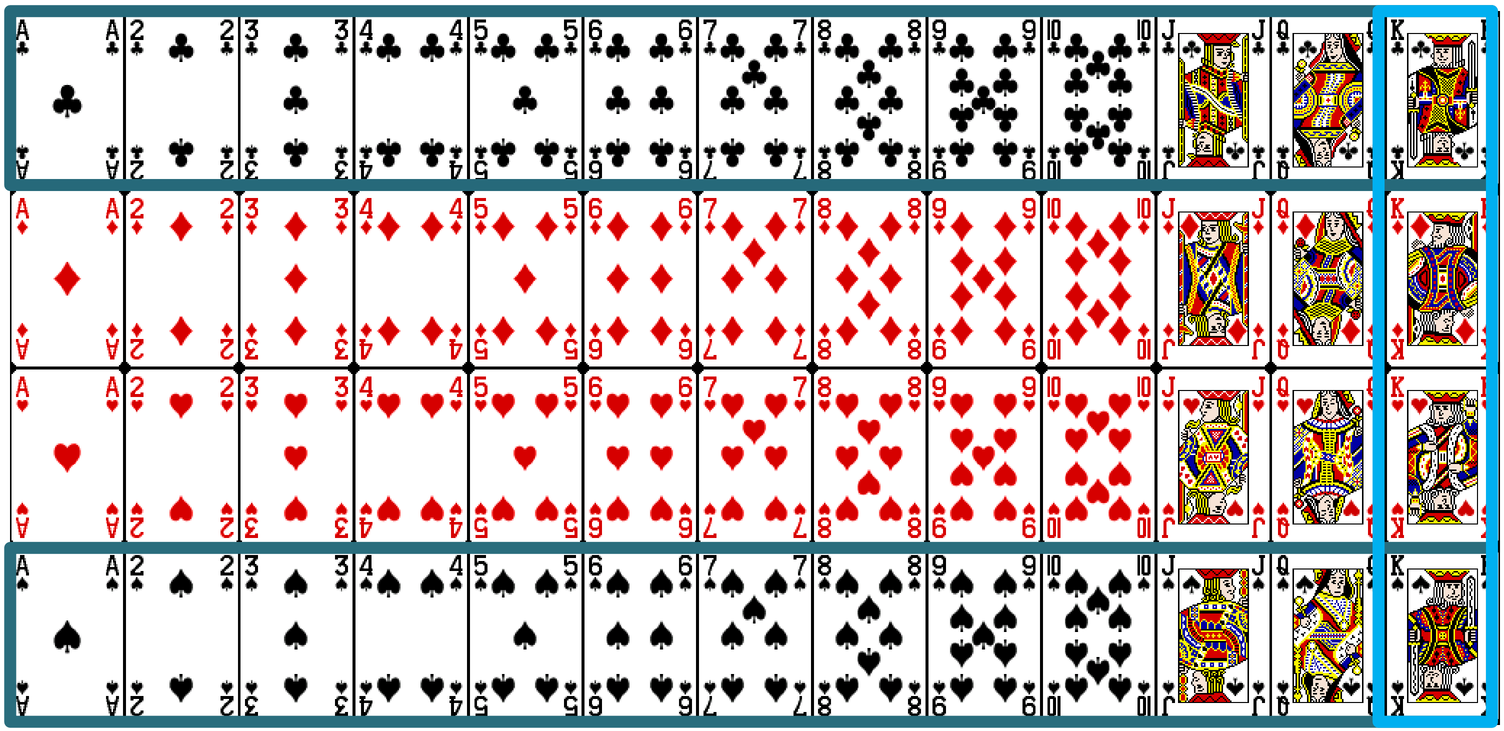
Example: Black & King

clubs

diamonds

hearts

spades

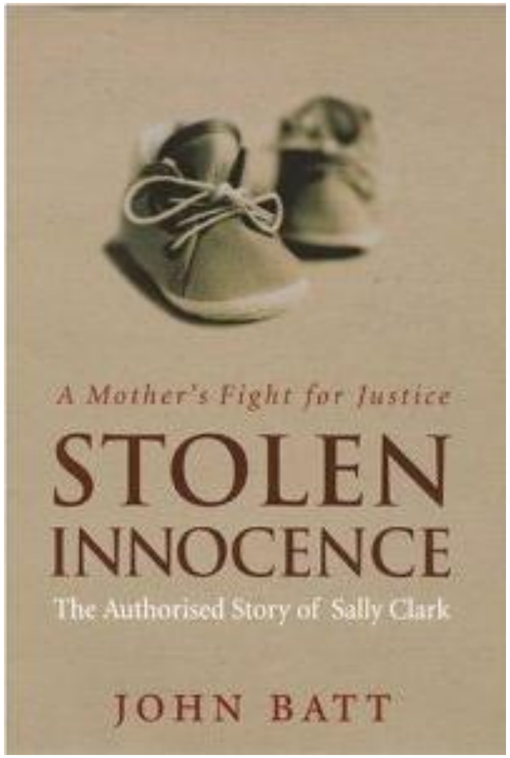


Sally Clark

[http://www.sallyclark.org.uk/]
[http://en.wikipedia.org/wiki/Sally_Clark]
[http://www.timesonline.co.uk/tol/comment/obituaries/article1533755.ece]

Sally Clark

- Falsely accused of the **murder of her two sons**.
  - Clark's first son died suddenly within a few weeks of his birth in 1996.
  - After her second son died in a similar manner, she was arrested in **1998** and tried for the murder of both sons.
- The case went to appeal, but the convictions and sentences were confirmed in 2000.
- Released in **2003** by Court of Appeal
- Wrongfully imprisoned for more than 3 years
- Never fully recovered from the effects of this appalling miscarriage of justice.



Misuse of statistics in the courts

- Her prosecution was controversial due to **statistical evidence**
- This evidence was presented by a medical expert witness Professor Sir Roy Meadow,

$$\left(\frac{1}{8500}\right)^2 \approx 10^{-8}$$

- Meadow testified that the frequency of sudden infant death syndrome (SIDS, or "cot death") in families having some of the characteristics of the defendant's family is 1 in 8500.
- He went on to **square** this figure to obtain a value of 1 in 73 million for the frequency of two cases of SIDS in such a family.

Royal Statistical Society

- “This approach is, in general, **statistically invalid**.”
- “It would only be valid if SIDS cases arose **independent**ly within families, an assumption that would need to be justified empirically. “
- “There are very strong a priori reasons for supposing that the assumption will be false.”
- “There may well be unknown genetic or environmental factors that predispose families to SIDS, so that **a second case within the family becomes much more likely**.”

[http://www.rss.org.uk]

Aftermath

- Clark's release in January 2003 prompted the Attorney General to order a review of hundreds of other cases.
- **Two other** women convicted of murdering their children had their convictions overturned and were released from prison.
- Trupti Patel, who was also accused of murdering her three children, was acquitted in June 2003.
- In each case, Roy Meadow had testified about the unlikelihood of multiple cot deaths in a single family.

How Juries Are Fooled by Statistics

- By Peter Donnelly

Professor of Statistical Science (Dept Statistics) at University of Oxford

@ 11:15-13:50 Disease Testing
@ 13:50-18:30 Sally Clark

http://www.youtube.com/watch?v=kLmzxmRcUTo
http://www.stats.ox.ac.uk/people/academic_staff/peter_donnelly

Prosecutor's Fallacy

- Aside from its invalidity, figures such as the 1 in 73 million are very easily misinterpreted.
- Some press reports at the time stated that this was the chance that the deaths of Sally Clark's two children were accidental.
- This (mis-)interpretation is a serious error of logic known as the **Prosecutor's Fallacy**.
- The jury needs to weigh up two competing explanations for the babies' deaths: 1) SIDS or 2) murder.
- Two deaths by SIDS or two murders are each quite unlikely, but one has apparently happened in this case.
- What matters is the relative likelihood of the deaths under each explanation, not just how unlikely they are under one explanation (in this case SIDS, according to the evidence as presented).

Independence among three events

- Can be checked via 2^3-3-1 = 4 conditions:

$$\text{Independence}\left\{\begin{array}{r} \left.\begin{array}{r} P(A\cap B)=P(A)P(B)\\ P(A\cap C)=P(A)P(C)\\ P(B\cap C)=P(B)P(C)\end{array}\right\}\text{Pairwise independence}\\ P(A\cap B\cap C)=P(A)P(B)P(C)\end{array}\right.$$

Remarks: Pairwise independence among the three events is defined by the first three conditions

Independence among four events

- Can be checked via 2^4-4-1 = 11 conditions:

$$P(A \cap B) = P(A)P(B)$$
$$P(A \cap C) = P(A)P(C)$$
$$P(A \cap D) = P(A)P(D)$$
$$P(B \cap C) = P(B)P(C)$$
$$P(B \cap D) = P(B)P(D)$$
$$P(C \cap D) = P(C)P(D)$$

Pairwise independence requires only these six conditions

$$P(B \cap C \cap D) = P(B)P(C)P(D)$$
$$P(A \cap C \cap D) = P(A)P(C)P(D)$$
$$P(A \cap B \cap D) = P(A)P(B)P(D)$$
$$P(A \cap B \cap C) = P(A)P(B)P(C)$$
$$P(A \cap B \cap C \cap D) = P(A)P(B)P(C)P(D)$$

Probability and Random Processes

ECS 315

Dr. Prapun Suksompong

prapun@siit.tu.ac.th

6.3 Bernoulli Trials

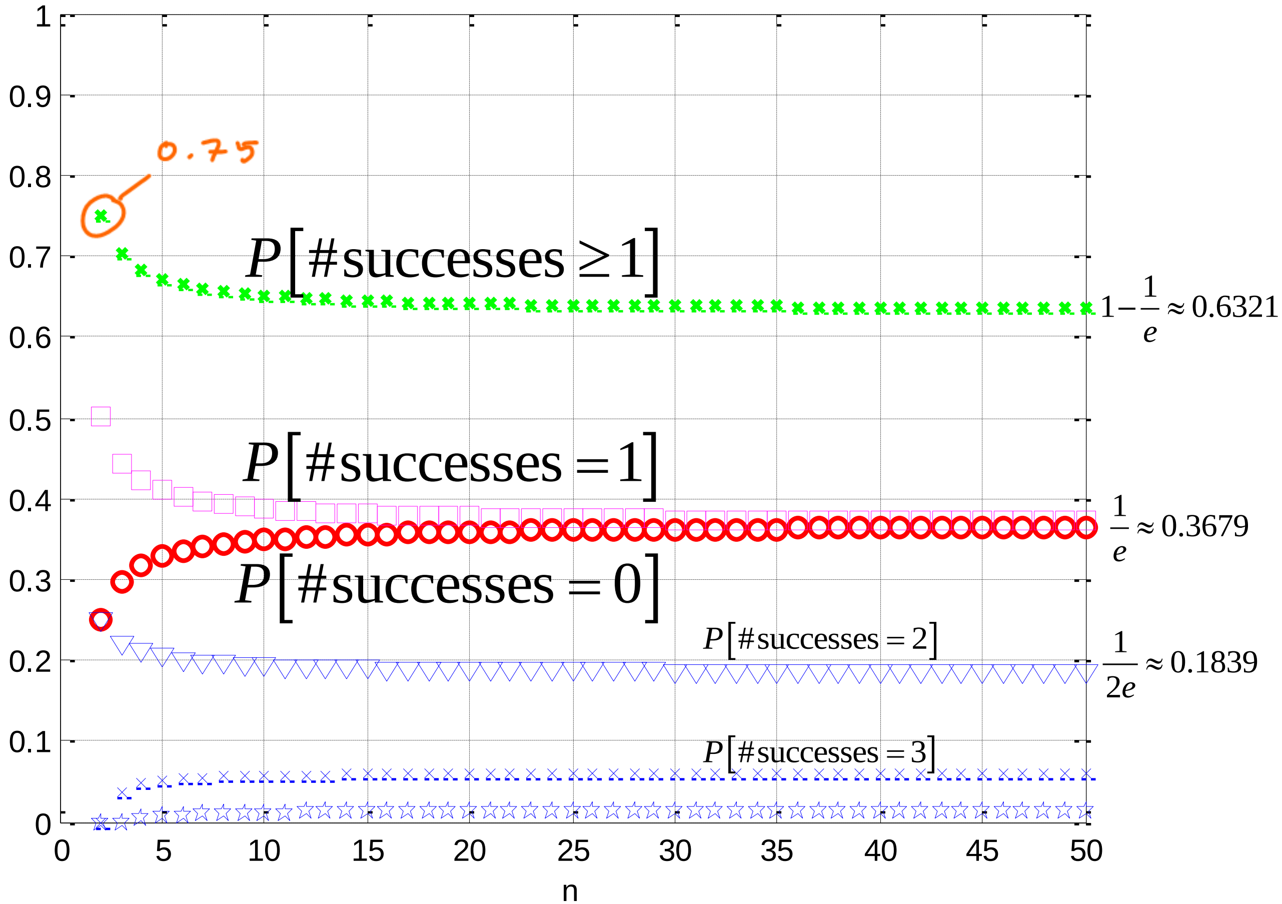
Office Hours:

BKD 3601-7

Monday	14:40-16:00
Friday	14:00-16:00

n Bernoulli trials

- Assume success probability = 1/n



Error Control Coding

- Repetition Code at Tx: Repeat the bit n times.
- Channel: Binary Symmetric Channel (BSC) with bit error probability p.
- Majority Vote at Rx