Probability and Random Processes
ECS 315

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Events-Based Probability Theory

Office Hours:
BKD 3601-7
Monday 14:40-16:00
Friday 14:00-16:00

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5 Foundation of Probability Theory

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Axioms of probability theory

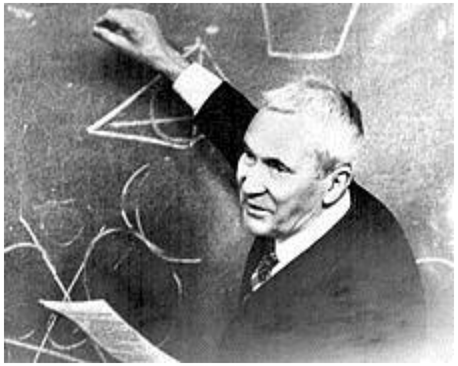
- Abstractly, a **probability measure** is a function that assigns numbers to events, which satisfies the following assumptions:

1. Nonnegativity: For any event A, $P(A) \geq 0$
2. Unit normalization: $P(\Omega) = 1$
3. If $A_1, A_2, \ldots,$ is an infinite sequence of (pairwise) **disjoint** events, then

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

Kolmogorov

- Andrey Nikolaevich Kolmogorov
- Soviet Russian mathematician
- Advanced various scientific fields
  - probability theory
  - topology
  - classical mechanics
  - computational complexity.



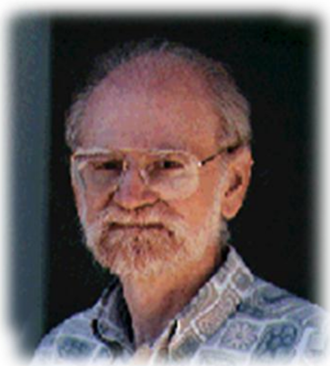
- 1922: Constructed a Fourier series that diverges almost everywhere, gaining international recognition.
- 1933: Published the book, **Foundations of the Theory of Probability**, laying the modern axiomatic foundations of probability theory and establishing his reputation as the world's leading living expert in this field.

I learn probability theory from


Eugene Dynkin


Philip Protter


Gennady Samorodnitsky


Terrence Fine


Xing Guo


Toby Berger


Rick Durrett

Not too far from Kolmogorov

You can be

the 4^{th}-generation

probability theorists

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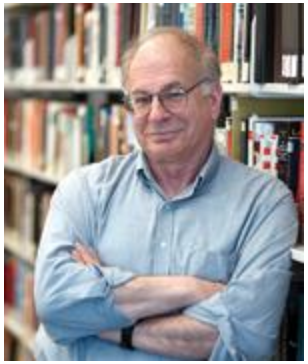
Event-Based Properties

Daniel Kahneman



- Daniel Kahneman
- Israeli-American **psychologist**
- 2002 **Nobel** laureate
  - In **Economics**
- Hebrew University, Jerusalem, Israel.
- Professor emeritus of psychology and public affairs at **Princeton** University's Woodrow Wilson School.
- With Amos **Tversky**, Kahneman studied and clarified the kinds of misperceptions of randomness that fuel many of the common fallacies.

[outspoken = given to expressing yourself freely or insistently]

K&T: Q1

Imagine a **woman** named **Linda**, **31** years old, **single**, **outspoken**, and very **bright**. In college she majored in **philosophy**. While a student she was deeply concerned with **discrimination** and **social justice** and participated in **antinuclear demonstrations**.

- K&T presented this description to a group of 88 subjects and asked them to **rank** the eight statements (shown on the next slide) on a scale of 1 to 8 according to their probability, with 1 representing the most probable and 8 the least.

[Daniel Kahneman, Paul Slovic, and Amos Tversky, eds., Judgment under Uncertainty: Heuristics and Biases (Cambridge: Cambridge University Press, 1982), pp. 90–98.]

[feminist = of or relating to or advocating equal rights for women]

K&T: Q1 - Results

- Here are the results - from most to least probable

Statement	*Average Probability Rank*
Linda is active in the feminist movement.	2.1
Linda is a psychiatric social worker.	3.1
Linda works in a bookstore and takes yoga classes.	3.3
Linda is a bank teller and is active in the feminist movement.	4.1
Linda is a teacher in an elementary school.	5.2
Linda is a member of the League of Women Voters.	5.4
Linda is a bank teller.	6.2
Linda is an insurance salesperson.	6.4

K&T: Q1 – Results (2)

- At first glance there may appear to be nothing unusual in these results: the description was in fact designed to be
  - representative of an active feminist and
  - unrepresentative of a bank teller or an insurance salesperson.

Most probable ↓ Least likely

Statement	*Average Probability Rank*
Linda is active in the feminist movement.	2.1 ←
Linda is a psychiatric social worker.	3.1
Linda works in a bookstore and takes yoga classes.	3.3
Linda is a bank teller and is active in the feminist movement.	4.1 ←
Linda is a teacher in an elementary school.	5.2
Linda is a member of the League of Women Voters.	5.4
Linda is a bank teller.	6.2 ←
Linda is an insurance salesperson.	6.4

K&T: Q1 – Results (3)

- Let's focus on just three of the possibilities and their average ranks.
- This is the order in which **85 percent** of the respondents ranked the three possibilities:

Statement	*Average Probability Rank*
Linda is active in the feminist movement.	2.1
Linda is a bank teller and is active in the feminist movement.	4.1
Linda is a bank teller.	6.2

- If nothing about this looks strange, then K&T have fooled you

K&T: Q1 - Contradiction

The probability that two events will both occur can never be greater than the probability that each will occur individually!

Statement	*Average Probability Rank*
Linda is active in the feminist movement.	2.1
Linda is a bank teller and is active in the feminist movement.	4.1
Linda is a bank teller.	6.2

K&T: Q2

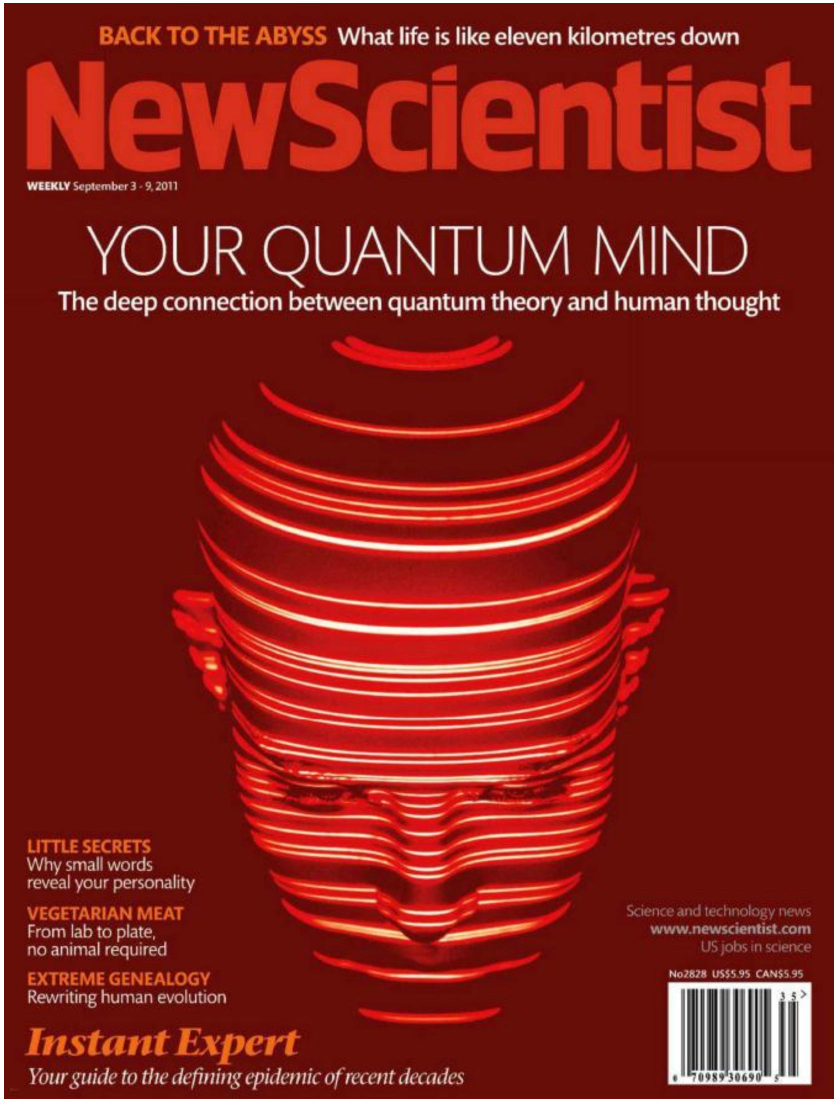
- K&T were not surprised by the result because they had given their subjects a large number of possibilities, and the connections among the three scenarios could easily have gotten lost in the shuffle.
- So they presented the description of Linda to another group, but this time they presented **only three possibilities**:
  - Linda is active in the feminist movement.
  - Linda is a bank teller and is active in the feminist movement.
  - Linda is a bank teller.

K&T: Q2 - Results

- To their surprise, **87 percent** of the subjects in this trial also **incorrectly** ranked the probability that "Linda is a bank teller and is active in the feminist movement" higher than the probability that "Linda is a bank teller".
- If the **details** we are given **fit our mental picture** of something, then the more details in a scenario, the more real it seems and hence the **more probable** we consider it to be
  - even though any act of adding less-than-certain details to a conjecture makes the conjecture less probable.
- Even **highly trained doctors** make this error when analyzing symptoms.
  - 91 percent of the doctors fall prey to the same bias.

[Amos Tversky and Daniel Kahneman, "Extensional versus Intuitive Reasoning: The Conjunction Fallacy in Probability Judgment," Psychological Review 90, no. 4 (October 1983): 293–315.]

Related Topic



- Page 34-37
- Tversky and Shafir @ Princeton University

Quantum minds

The fuzziness and weird logic of the way particles behave applies surprisingly well to how humans think. Mark Buchanan finds the "you" in quantum

THE quantum world defies the rules of ordinary logic. Particles routinely occupy two or more places at the same time and don't even have well-defined properties until they are measured. It's all strange, yet true – quantum theory is the most accurate scientific theory ever tested and its mathematics is perfectly suited to the weirdness of the atomic world.

Yet that mathematics actually stands on its own, quite independent of the theory. Indeed, much of it was invented well before quantum theory even existed, notably by German mathematician David Hilbert. Now, it's beginning to look as if it might apply to a lot more than just quantum physics, and quite possibly even to the way people think.

Human thinking, as many of us know, often fails to respect the principles of classical logic. We make systematic errors when reasoning with probabilities, for example. Physicist Diederik Aerts of the Free University of Brussels, Belgium, has shown that these errors actually make sense within a wider logic based on quantum mathematics. The same logic also seems to fit naturally with how people link concepts together, often on the basis of loose associations and blurred boundaries. That means search algorithms based on quantum logic could uncover meanings in masses of text more efficiently than classical algorithms.

It may sound preposterous to imagine that the mathematics of quantum theory has something to say about the nature of human thinking. This is not to say there is anything quantum going on in the brain, only that "quantum" mathematics really isn't owned by physics at all, and turns out to be better than classical mathematics in capturing the fuzzy and flexible ways that humans use ideas. than the one dictated by classical logic," says Aerts. "The mathematics of quantum theory turns out to describe this quite well."

It's a finding that has kicked off a burgeoning field known as "quantum interaction", which explores how quantum theory can be useful in areas having nothing to do with physics, ranging from human language and cognition to biology and economics. And it's already drawing researchers to major conferences.

One thing that distinguishes quantum from classical physics is how probabilities work. Suppose, for example, that you spray some particles towards a screen with two slits in it, and study the results on the wall behind (see diagram, page 36). Close slit B, and particles going through A will make a pattern behind it. Close A instead, and a similar pattern will form behind slit B. Keep both A and B open and the pattern you should get – ordinary physics and logic would suggest – should be the sum of these two component patterns.

But the quantum world doesn't obey. When electrons or photons in a beam pass through the two slits, they act as waves and produce an interference pattern on the wall. The pattern with A and B open just isn't the sum of the two patterns with either A or B open alone, but something entirely different – one that varies as light and dark stripes.

Such interference effects lie at the heart of many quantum phenomena, and find a natural description in Hilbert's mathematics. But the phenomenon may go well beyond physics, and one example of this is the violation of what logicians call the "sure thing" principle. This is the idea that if you prefer one action over another in one situation – coffee over tea in situation A, say, when it's before noon – and you prefer the same thing in the opposite situation – coffee over tea in situation B, when it's after noon – then you should have the same preference when you don't know the situation: that is, coffee over tea when you don't know what time it is.

Remarkably, people don't respect this rule. In the early 1990s, for example, ›

K&T: Q3

- Which is greater:
  - the number of six-letter English words having "n" as their fifth letter
    or
  - the number of six-letter English words ending in "-ing"?
- Most people choose the group of words ending in "ing". Why? Because words ending in "-ing" are easier to think of than generic six letter words having "n" as their fifth letter.
- The group of six-letter words having "n" as their fifth letter words includes all six-letter words ending in "-ing".
- Psychologists call this type of mistake the **availability bias**
  - In reconstructing the past, we give unwarranted importance to memories that are most vivid and hence most available for retrieval.
    [Amos Tversky and Daniel Kahneman, "Availability: A Heuristic for Judging Frequency and Probability," Cognitive Psychology 5 (1973): 207–32.]

Misuse of probability in law

- It is not uncommon for experts in **DNA analysis** to testify at a criminal trial that a DNA sample taken from a crime scene matches that taken from a suspect.
- How certain are such matches?
- When DNA evidence was first introduced, a number of experts testified that false positives are impossible in DNA testing.
- Today DNA experts regularly testify that the odds of a random person's matching the crime sample are less than **1 in 1 million** or **1 in 1 billion**.
- In Oklahoma a court sentenced a man named Timothy Durham to prison even though **eleven witnesses** had placed him in another state at the time of the crime.

[Mlodinow, 2008, p 36-37]

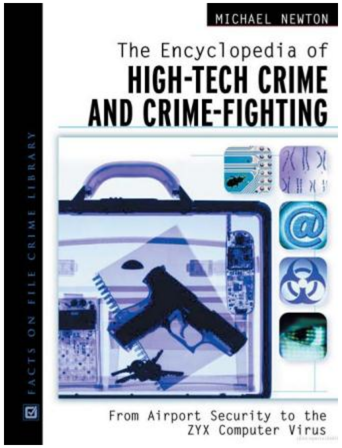
Lab/Human Error

- There is **another stat**istic that is often **not presented** to the jury, one having to do with the fact that **labs make errors**, for instance, in collecting or handling a sample, by accidentally mixing or swapping samples, or by misinterpreting or incorrectly reporting results.
- Each of these errors is rare but not nearly as rare as a random match.
- The Philadelphia City Crime Laboratory admitted that it had swapped the reference sample of the defendant and the victim in a rape case
- A testing firm called Cellmark Diagnostics admitted a similar error.

[Mlodinow, 2008, p 36-37]

Timothy Durham's case

- It turned out that in the initial analysis the lab had failed to completely separate the DNA of the rapist and that of the victim in the fluid they tested, and the combination of the victim's and the rapist's DNA produced a positive result when compared with Durham's.
- A later retest turned up the error, and Durham was released after spending nearly **four years** in prison.



[Mlodinow, 2008, p 36-37]

DNA-Match Error + Lab Error

- Estimates of the error rate due to human causes vary, but many experts put it at around 1 percent.
- Most jurors assume that given the two types of error—the **1 in 1 billion** accidental match and the **1 in 100 lab-error match**—the overall error rate must be somewhere in between, say 1 in 500 million, which is still for most jurors **beyond a reasonable doubt**.

[Mlodinow, 2008, p 36-37]

Wait!...

- Even if the DNA match error was extremely accurate + Lab error is very small,
- there is also another probability concept that should be taken into account.
- More about this later.
- Right now, back to notes for more properties of probability measure.