Probability and Random Processes

ECS 315

Dr. Prapun Suksompong

prapun@siit.tu.ac.th

Multiple Random Variables

Office Hours:

BKD 3601-7

Monday	**14:40-16:00**
Friday	**14:00-16:00**

Review (1.1)

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$$P_{X,Y} \quad \begin{array}{c|ccccc} x \backslash y & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0.1 & 0.1 & 0 & 0 & 0 \\ 3 & 0.1 & 0 & 0 & 0.1 & 0 \\ 4 & 0 & 0.1 & 0.2 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0.3 \end{array}$$

- Find $P[X + Y < 7]$

Step 1: Find the pairs (x,y) that satisfy the condition "$x+y < 7$"

One way to do this is to first construct the matrix of $x+y$.

$$x+y \quad \begin{array}{c|ccccc} x \backslash y & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 7 & 8 & 9 & 10 \\ 6 & 8 & 9 & 10 & 11 & 12 \end{array}$$

Review (1.2)

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$$P_{X,Y} \quad \begin{array}{c|ccccc} x \backslash y & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0.1 & 0.1 & 0 & 0 & 0 \\ 3 & 0.1 & 0 & 0 & 0.1 & 0 \\ 4 & 0 & 0.1 & 0.2 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0.3 \end{array}$$

- Find $P[X+Y<7]$

Step 2: Add the corresponding probabilities from the joint pmf (matrix)

$$x+y \quad \begin{array}{c|ccccc} x \backslash y & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 7 & 8 & 9 & 10 \\ 6 & 8 & 9 & 10 & 11 & 12 \end{array}$$

$$P[X+Y<7] = 0.1+0.1+0.1 = 0.3$$

Review (1.3)

- Formula-wise:

$$P[X+Y<7] = \underbrace{\sum_{\substack{(x,y)\\ \underbrace{x+y<7}_{\text{Step 1}}}}}_{\text{Step 2}} p_{X,Y}(x,y)$$

$$\begin{array}{c|ccccc} {}_{x}\backslash^{y} & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 7 & 8 & 9 & 10 \\ 6 & 8 & 9 & 10 & 11 & 12 \end{array} \qquad x+y$$

For our example, only

$$(x,y) \in \left\{ \begin{matrix} (1,2),(1,3),(1,4), \\ (1,5),(3,2),(3,3), \\ (4,2) \end{matrix} \right\}$$

satisfy the condition

- Alternative way to write this:

$$P[X+Y<7] = \sum_{x} \sum_{\substack{y \\ x+y<7}} p_{X,Y}(x,y) = \sum_{y} \sum_{\substack{x \\ x+y<7}} p_{X,Y}(x,y)$$

Review (2.1)

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$$P_{X,Y} \quad \begin{array}{c|c} {}_{x}\backslash^{y} & \begin{array}{ccccc} 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 3 \\ 4 \\ 6 \end{array} & \left[\begin{array}{ccccc} 0.1 & 0.1 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0.1 & 0 \\ 0 & 0.1 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3 \end{array}\right] \end{array}$$

- Find $P[X+Y=7]$

$P[X+Y=7] = 0.1$

$$x+y \quad \begin{array}{c|c} {}_{x}\backslash^{y} & \begin{array}{ccccc} 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 3 \\ 4 \\ 6 \end{array} & \left[\begin{array}{ccccc} 3 & 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 & 9 \\ 6 & 7 & 8 & 9 & 10 \\ 8 & 9 & 10 & 11 & 12 \end{array}\right] \end{array}$$

Review (2.2)

- Formula-wise:

Step 2

$$P[X+Y=7]=\sum_{\substack{(x,y)\\x+y=7}} p_{X,Y}(x,y)$$

Step 1

For our example, only $(x,y) \in \{(1,6),(3,4),(4,3)\}$ satisfy the condition

$$x+y \quad \begin{array}{c|c} {}_{x}\diagdown^{y} & \begin{matrix} 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 & 9 \\ 6 & 7 & 8 & 9 & 10 \\ 8 & 9 & 10 & 11 & 12 \end{bmatrix} \end{array}$$

- Other ways to write (and think about) this:

$$P[X+Y=7]=\sum_{x}\sum_{\substack{y\\x+y=7}} p_{X,Y}(x,y)=\sum_{x} p_{X,Y}(x,7-x)$$

$$=\sum_{y}\sum_{\substack{x\\x+y=7}} p_{X,Y}(x,y)=\sum_{y} p_{X,Y}(7-y,y)$$

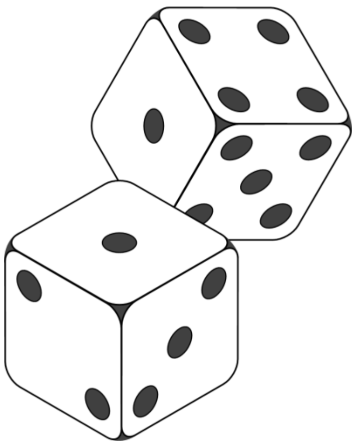
Z = g(X,Y)

- In this section, we define a new random variable Z using the two random variables X and Y.
- For example, $Z = X + Y$.
  - Technically, this means $Z(\omega) = X(\omega) + Y(\omega)$.
- Because Z is a random variable, we can also find its pmf.
- Recall that $p_Z(z) = P[Z = z]$.
- However, $Z = X + Y$. So, $p_Z(z) = P[X + Y = z]$.
- For example, $p_Z(7) = P[X + Y = 7]$.
- Therefore, we can use the same technique that we just reviewed to find $p_Z(z)$ at each z:

$$p_Z(z) = P[X + Y = z] = \sum_x p_{X,Y}(x, z - x) = \sum_y p_{X,Y}(z - y, y)$$

Sum of Two dice

- Assume that the two dice are fair and independent.



DICE CHART

ROLL		PROBABILITY
2		1/36
3		2/36
4		3/36
5		4/36
6		5/36
7		6/36
8		5/36
9		4/36
10		3/36
11		2/36
12		1/36

Sum of two indp random variables

- = convolution of their pmf

```
p =

    0.1667    0.1667    0.1667    0.1667    0.1667    0.1667

>> conv(p,p)

ans =

  Columns 1 through 9

    0.0278    0.0556    0.0833    0.1111    0.1389    0.1667    0.1389    0.1111    0.0833

  Columns 10 through 11

    0.0556    0.0278

>> sym(conv(p,p))

ans =

[ 1/36, 1/18, 1/12, 1/9, 5/36, 1/6, 5/36, 1/9, 1/12, 1/18, 1/36]
```