Name........................................................ID..........................Section.......1.......Seat No...........

# Sirindhorn International Institute of Technology
# Thammasat University

## Midterm Examination: Semester 1 / 2017

Course Title: ECS315 (Probability and Random Processes)

Instructor: Asst. Prof. Dr.Prapun Suksompong

Date/Time: October 5, 2017 / 15:00 - 17:00

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**Instructions:**

- This examination has ....7.... pages (including this cover page).
- Conditions of Examination:
  ............Closed book
  (No dictionary, ☐ No calculator ☑ Calculator (e.g. FX-991) allowed)
  ............Open book
  .....✓.... **Semi-Closed book** (......1.......sheet(s) ☑ 1 page ☐ both sides of A4 paper note)
  This sheet must be hand-written.
  Do not modify (,e.g., add/underline/highlight) content on the sheet inside the exam room.
  It should be **<u>submitted with the exam</u>**.
  Other requirements are specified on the course web site. (-10 pt if not following the requirements.)
- **<u>Read</u> these instructions and the questions carefully**.
- Students are not allowed to be out of the examination room during examination. Going to the restroom may result in score deduction.
- Turn off all communication devices and place them with other personal belongings in the area designated by the proctors or outside the test room.
- Write your name, student ID, section, and seat number clearly in the spaces provided on the top of this sheet. Then, write your **first name and the <u>last three digits</u> of your ID** in the spaces provided on the top of each page of your examination paper, starting from page 2.
- <u>The back of each page</u> will <u>not</u> be graded; it can be used for calculations of problems that do not require explanation.
- The examination paper is not allowed to be taken out of the examination room. Also, <u>do not remove the staple</u>. Violation may result in score deduction.
- Unless instructed otherwise**, *<u>write down all the steps</u>*** that you have done to obtain your answers.
  - When applying formula(s), state clearly which formula(s) you are applying before plugging-in numerical values.
  - You may not get any credit even when your final answer is correct without showing how you get your answer.
  - Formula(s) not discussed in class can be used. However, derivation must also be provided.
  - **<u>Exceptions</u>**:
    - Problems that are labeled with “ENRPr” (Explanation is not required for this problem.)
    - Parts that are labeled with “ENRPa” (Explanation is not required for this part.)
    - These problems/parts are graded solely on your answers. There is no partial credit and it is not necessary to write down your explanation. Usually, spaces (boxes or cells in a table or rows of dashes) will be provided for your answers. “WACSP” stands for “write your answer(s) in the corresponding space(s) provided”.
- When not explicitly stated/defined, all notations and definitions follow ones given in lecture.
- Some points are reserved for *accuracy* of the answers and also for reducing answers into their *simplest* forms. Watch out for roundoff error.
- Points marked with * indicate challenging problems.
- Do not cheat. Do not panic. **Allocate your time wisely.**
- Don’t forget to submit your fist online self-evaluation form by the end of today.

**Problem 1.** (10 pt) [ENRPr] For each of the sets provided in the first column of the table below, indicate (by putting a Y(es) or an N(o) in the appropriate cells of the table) whether it is "finite", "infinite", "countably infinite", "uncountable". Note that $\mathbb{R}$ is the set of all real numbers.

| Sets | Finite | Infinite | Countably Infinite | Uncountable |
|---|---|---|---|---|
| $\mathbb{R}$ | | | | |
| $\{\pi, 2\pi\}$ | | | | |
| $[1,3] \cap [2,4]$ | | | | |
| $[1,2] \cap [3,4]$ | | | | |
| the set of all real-valued $x$ satisfying $\sin(x) = x$ | | | | |

**Problem 2.** (5 pt)

(a) (4 pt) Calculate the following quantities:

(i) (1 pt) $3!$

(ii) (1 pt) $\binom{6}{3}$

(iii) (1 pt) $(6)_3$

(iv) (1 pt) $\binom{6}{1,2,3}$

(b) (1* pt) Without the help of your calculator, show how to simplify the following expression

$$2 \times \binom{1000}{2} + 3 \times \binom{1000}{3} + \cdots + 1000 \times \binom{1000}{1000}$$

**Problem 3.** (4 pt) [ENRPr] Suppose we sample 4 objects from a collection of 6 distinct objects. Calculate the number of different possibilities when

(a) the sampling is unordered and performed with replacement

(b) the sampling is unordered and performed without replacement

(c) the sampling is ordered and performed with replacement

(d) the sampling is ordered and performed without replacement

**Problem 4.** ($2 \times 5 = 10$ pt)

(a) Calculate the number of different results when we permute the letters in each of the following collections:

(i) ABC

(ii) AABBCC

(b) In the expansion of $(x + y)^{10}$, find the coefficient of the following terms:

(i) $x^2y^8$

(ii) $x^4y^7$

(c) Calculate the number of quadruples $(x_1, x_2, x_3, x_4)$ satisfying

$$x_1 + x_2 + x_3 + x_4 = 10.$$

Assume all variables are nonnegative integers.

**Problem 5.** (8 pt) A Web ad can be designed from three different colors (red, green, blue), four font types (Helvetica, Frutiger, Myriad Pro, Avenir Std), and five font sizes. A specific design is randomly generated by the Web server each time that you visit the site.

(a) (2 pt) How many different designs are possible?

(b) (4 pt) [ENRPa] Suppose you visit the site.
Let $A$ denote the event that the design color is blue and
let $B$ denote the event that the font size is not the largest one.
Find the following quantities:

$P(A) =$ $\qquad$ $P(B) =$

$P(A \cap B) =$ $\qquad$ $P(A \cup B^c) =$

(c) (2 pt) If you visit the site three times, what is the probability that you will not see the same design?

**Problem 6.** (8 pt) [ENRPr] Consider a random experiment whose sample space is $\{a, b, c, d\}$ with probabilities 0.2, 0.2, 0.3, 0.3, respectively.
Let $A = \{a, b, c\}$, $B = \{c, d\}$, and $C = \{a, c\}$.
Find the following probabilities.

$P(A) =$ $\qquad\qquad$ $P(B^c) =$

$P(A \cap B) =$ $\qquad\qquad$ $P(A \cup B) =$

$P(A|B) =$ $\qquad\qquad$ $P(A|B^c) =$

$P(A^c|B) =$ $\qquad\qquad$ $P(A \cap B|C) =$

**Problem 7.** (8 pt) [ENRPr] In each of the parts below, find $P(A)$, $P(B)$, and $P(A \cap B)$. Then, determine (by putting a Y(es) or an N(o) in the provided space) whether events $A$ and $B$ are independent.

(a) $P(A^c \cap B^c) = 0.1$, $P(A \cap B^c) = 0.2$, and $P(A^c \cap B) = 0.3$.

$P(A) =$ _____, $P(B) =$ _____, $P(A \cap B) =$ _____;

$A \perp\!\!\!\perp B$? _____

(b) $P(A \cup B) = 0.43$, $P(A \cup B^c) = 0.62$, $P(A^c \cup B) = 0.97$.

$P(A) =$ _____, $P(B) =$ _____, $P(A \cap B) =$ _____;

$A \perp\!\!\!\perp B$? _____

**Problem 8.** ($2 \times 4 + 1^* = 9$ pt) [Digital Communication] A certain binary-symmetric channel has a crossover probability (bit-error rate) of 0.4. Assume bit errors occur independently. Your answers for parts (a) and (b) should be of the form X.XXXX.

(a) Suppose we input bit sequence "1010101" into this channel.

(i) What is the probability that the output is "1000001"?

(ii) What is the probability that exactly 4 bits are in error at the channel output ?

(iii) What is the probability that there is at least one bit error at the channel output?

(b) Suppose we keep inputting bits into this channel. What is the probability that the *first* bit error at the output occurs on the fourth bit?

(c) Suppose the input bits are generated by flipping a fair coin 7 times. Heads and tails are represented by 1 and 0, respectively.
Let $A$ be the event that the output of the channel is "1000001".
Let $B_1$ be the event that the input of the channel is "1100011".
Let $B_2$ be the event that the input of the channel is "1011101".
Compare $P(B_1|A)$ and $P(B_2|A)$. (Which one is larger? Explain.)

**Problem 9.** (3 pt) You have four coins in your pocket, three fair ones but the fourth biased with probability of heads $\frac{1}{5}$. One coin selected at random drops to the floor, landing heads up. How likely is it that it is one of the fair coins?
(Make sure that your events are clearly defined.)

**Problem 10.** (1* pt) Suppose

$$P(A) = \frac{1}{2}, \quad P(B) = \frac{1}{3}, \quad P(C) = \frac{1}{4}, \quad P(A \cup B \cup C) = \frac{3}{4}.$$

Furthermore,

$$A \perp\!\!\!\perp B, \quad B \perp\!\!\!\perp C, \quad A \perp\!\!\!\perp C.$$

Are $A, B$, and $C$ independent?

**Problem 11.** (1 pt)

(a) Do not forget to submit your study sheet with your exam.

(b) Make sure that you write your name and ID on every page. (Read the instruction on the cover page.)

(c) The online self-evaluation form is due by the end of today.