Name ________________________ ID3 _ _ _

ECS 315: Probability and Random Processes **2018/1**

HW 4 — Due: September 18, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 4 pages.

(b) (1 pt) Work and write your answers **directly on these sheets** (not on another blank sheet of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

Problem 1. Continue from Problem 2 in HW3.

Recall that, there, we consider a random experiment whose sample space is $\{a, b, c, d, e\}$ with probabilities 0.1, 0.1, 0.2, 0.4, and 0.2, respectively. Let A denote the event $\{a, b, c\}$, and let B denote the event $\{c, d, e\}$. Find the following probabilities.

(a) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

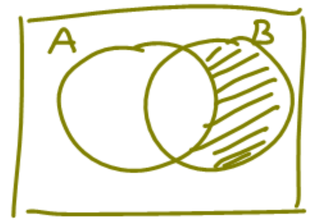
(b) $P(B|A) = \frac{P(A \cap B)}{P(A)}$

(c) $P(B|A^c) = \frac{P(A^c \cap B)}{P(A^c)} = \frac{P(B \setminus A)}{P(A^c)}$

Problem 2.

(a) Suppose that $P(A|B) = 0.4$ and $P(B) = 0.5$ Determine the following:

(i) $P(A \cap B)$ $= P(B)P(A|B) = 0.5 \times 0.4 = 0.2$



$B \setminus A$

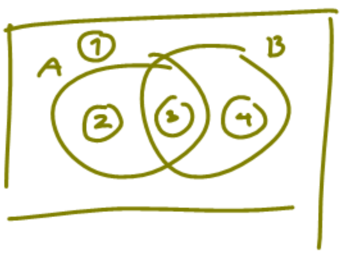
(ii) $P(A^c \cap B)$ $= P(B)P(A^c|B) = 0.5 \times 0.6 = 0.3$

$= P(B) - P(A \cap B) = 0.5 - 0.2 = 0.3$

$P(A^c|B) = 1 - P(A|B) = 1 - 0.4 = 0.6$

[Montgomery and Runger, 2010, Q2-105]

(b) Suppose that $P(A|B) = 0.2$, $P(A|B^c) = 0.3$ and $P(B) = 0.8$ What is $P(A)$? [Montgomery and Runger, 2010, Q2-106]



$P(A) = P(A \cap B) + P(A \cap B^c)$

$= P(A|B)P(B) + P(A|B^c)P(B^c)$

Problem 3. Suppose that for the general population, 1 in 5000 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive (+) or negative (-) response. Suppose the test gives the correct answer 99% of the time.

(a) What is $P(-|H)$, the conditional probability that a person tests negative given that the person does have the HIV virus?

(b) What is $P(H|+)$, the conditional probability that a randomly chosen person has the HIV virus given that the person tests positive?

Problem 4. Due to an Internet configuration error, packets sent from New York to Los Angeles are routed through El Paso, Texas with probability 3/4. Given that a packet is routed through El Paso, suppose it has conditional probability 1/3 of being dropped. Given that a packet is not routed through El Paso, suppose it has conditional probability 1/4 of being dropped. [Gubner, 2006, Ex.1.20]

(a) Find the probability that a packet is dropped.
Hint: Use total probability theorem.

(b) Find the conditional probability that a packet is routed through El Paso given that it is not dropped.
Hint: Use Bayes' theorem.

Extra Questions

Here are some optional questions for those who want more practice.

Problem 5. Someone has rolled a fair dice twice. Suppose he tells you that "one of the rolls turned up a face value of six". What is the probability that the other roll turned up a six as well? [Tijms, 2007, Example 8.1, p. 244]

Hint: Note the followings:

- The answer is not $\frac{1}{6}$.
- Although there is no use of the word "given" or "conditioned on" in this question, the probability we seek is a conditional one. We have an extra piece of information because we know that the event "one of the rolls turned up a face value of six" has occurred.
- The question says "one of the rolls" without telling us which roll (the first or the second) it is referring to.

Problem 6.

(a) Suppose that $P(A|B) = 1/3$ and $P(A|B^c) = 1/4$. Find the range of the possible values for $P(A)$.

(b) Suppose that C_1, C_2, and C_3 partition Ω. Furthermore, suppose we know that $P(A|C_1) = 1/3$, $P(A|C_2) = 1/4$ and $P(A|C_3) = 1/5$. Find the range of the possible values for $P(A)$.